18 ATMOSPHERIC BOUNDARY LAYER

Contents

Sunrise, sunset, sunrise. The daily cycle of radiative heating and cooling of the Earth's surface during clear skies causes a daily cycle of sensible and latent heat fluxes between the land and the air. These fluxes modify the bottom of the troposphere — a layer called the **atmospheric boundary layer** (ABL) because it is influenced by the bottom boundary of the atmosphere (Fig. 18.1).

The ABL experiences a **diurnal** (daily) cycle of temperature, humidity, wind, and pollution variations in response to the varying surface fluxes. Turbulence is ubiquitous in the ABL, and is one of the causes of the unique nature of the ABL.

Because the boundary layer is where we live, where our crops are grown, and where we conduct our commerce, we have become familiar with its daily cycle. We perhaps forget that this cycle is not experienced by the rest of the atmosphere above the ABL. This chapter examines the formation and unique characteristics of the ABL.

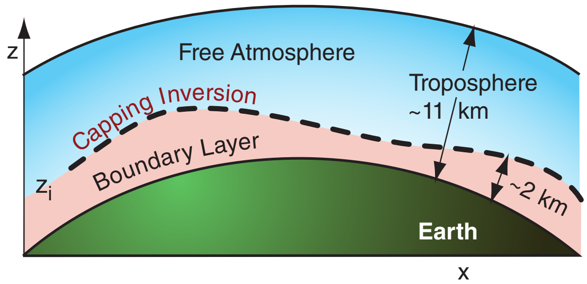


Figure 18.1
Location of the boundary layer, with top at z_i.

18.1. STATIC STABILITY — A REVIEW

18.1.1. Explanation

Static stability controls the formation of the ABL, and affects ABL wind and temperature profiles. Here is a quick review of information from the Atmospheric Stability chapter.

If a small blob of air (i.e., an **air parcel**) is warmer than its surroundings at the same height or pressure, the parcel is positively buoyant and rises. If cooler, it is negatively buoyant and sinks. A parcel

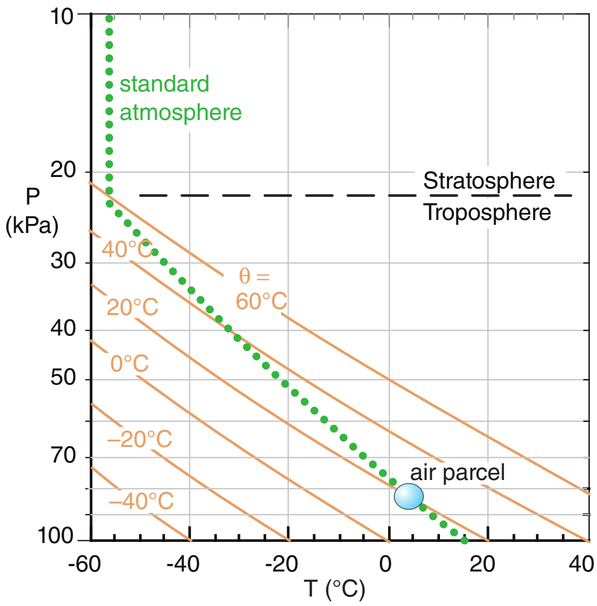


Figure 18.2
Standard atmosphere (dotted green line) plotted on a thermodynamic diagram (emagram). The circle represents a hypothetical air parcel. Diagonal tan lines are dry adiabats.

Sample Application

Find the vertical gradient of potential temperature in the troposphere for a standard atmosphere.

Find the Answer

Given: $\Delta T/\Delta z = -6.5°C\ km^{-1}$ from Chapter 1 eq. (1.16)
Find: $\Delta\theta/\Delta z = ?\ °C\ km^{-1}$

Use eq. (3.11) from the Heat chapter:

$$\theta(z) = T(z) + \Gamma_d \cdot z \qquad (3.11)$$

where the dry adiabatic lapse rate is $\Gamma_d = 9.8°C\ km^{-1}$
Apply this at heights z_1 and z_2, and then subtract the z_1 equation from the z_2 equation:

$$\theta_2 - \theta_1 = T_2 - T_1 + \Gamma_d \cdot (z_2 - z_1)$$

Divide both sides of the equation by $(z_2 - z_1)$. Then define $(z_2 - z_1) = \Delta z$, $(T_2 - T_1) = \Delta T$, and $(\theta_2 - \theta_1) = \Delta\theta$ to give the algebraic form of the answer:

$$\Delta\theta/\Delta z = \Delta T/\Delta z + \Gamma_d$$

This eq. applies to <u>any</u> vertical temperature profile.

If we plug in the temperature profile for the standard atmosphere:

$$\Delta\theta/\Delta z = (-6.5°C\ km^{-1}) + (9.8°C\ km^{-1}) = \underline{\mathbf{3.3°C\ km^{-1}}}$$

Check: Units OK. Agrees with Fig. 18.2, where θ increases from 15°C at the surface to 51.3°C at 11 km altitude, which gives (51.3–15°C)/(11km) = 3.3°C km^{-1}.
Exposition: θ gradually <u>in</u>creases with height in the troposphere, which as we will see tends to gently oppose vertical motions. Although the standard-atmosphere (an engineering specification similar to an average condition) troposphere is statically stable, the real troposphere at any time and place can have layers that are statically stable, neutral, or unstable.

with the same temperature as its surrounding environment experiences zero buoyant force.

Figure 18.2 shows the standard atmosphere from Chapter 1, plotted on a thermodynamic diagram from the Stability chapter. Let the standard atmosphere represent the environment or the background air. Consider an air parcel captured from one part of that environment (plotted as the circle). At its initial height, the parcel has the same temperature as the surrounding environment, and experiences no buoyant forces.

To determine static stability, you must ask what would happen to the air parcel if it were forcibly displaced a small distance up or down. When moved from its initial capture altitude, the parcel temperature could differ from that of the surrounding environment, thereby causing buoyant forces.

If the buoyant forces on a displaced air parcel push it back to its starting altitude, then the environment is said to be **statically stable**. In the absence of any other forces, statically stable air flow is **laminar**. Namely, it is smooth and non-turbulent.

However, if the displaced parcel is pulled further away from its starting point by buoyancy, the portion of the atmosphere through which the air parcel continues accelerating is classified as **statically unstable**. Unstable regions are **turbulent** (gusty).

If the displaced air parcel has a temperature equal to that of its new surroundings, then the environment is **statically neutral**.

When an air parcel moves vertically, its temperature changes adiabatically, as described in previous chapters. Always consider such adiabatic temperature change before comparing parcel temperature to that of the surrounding environment. The environment is usually assumed to be **stationary**, which means it is relatively unchanging during the short time it takes for the parcel to rise or sink.

If an air parcel is captured at P = 83 kPa and T = 5°C (as sketched in Fig. 18.2), and is then forcibly lifted dry adiabatically, it cools following the θ = 20°C adiabat (one of the thin diagonal lines in that figure). If lifted to a height where the pressure is P = 60 kPa, its new temperature is about T = –20°C.

This air parcel, being colder than the environment (thick dotted line in Fig. 18.2) at that same height, feels a downward buoyant force toward its starting point. Conversely, if displaced downward from its initial height, the parcel would be warmer than its surroundings at its new height, and would feel an upward force toward its starting point.

Air parcels captured from any initial height in the environment of Fig. 18.2 always tend to return to their starting point. Therefore, <u>the standard atmosphere is statically stable</u>. This stability is critical for ABL formation.

18.1.2. Rules of Thumb for Stability in the ABL

Because of the daily cycle of radiative heating and cooling, there is a daily cycle of static stability in the ABL. ABL static stability can be anticipated as follows, without worrying about air parcels for now.

Unstable air adjacent to the ground is associated with light winds and a surface that is warmer than the air. This is common on sunny days in fair-weather. It can also occur when cold air blows over a warmer surface, day or night. In unstable conditions, thermals of warm air rise from the surface to heights of 200 m to 4 km, and turbulence within this layer is vigorous.

At the other extreme are **stable** layers of air, associated with light winds and a surface that is cooler than the air. This typically occurs at night in fair-weather with clear skies, or when warm air blows over a colder surface day or night. Turbulence is weak or sometimes nonexistent in stable layers adjacent to the ground. The stable layers of air are usually shallow (20 - 500 m) compared to the unstable daytime cases.

In between these two extremes are **neutral** conditions, where winds are moderate to strong and there is little heating or cooling from the surface. These occur during overcast conditions, often associated with bad weather. They also occur in a part of the ABL called the residual layer, defined later.

18.2. BOUNDARY-LAYER FORMATION

18.2.1. Tropospheric Constraints

Because of buoyant effects, the vertical temperature structure of the troposphere limits the types of vertical motion that are possible. The standard atmosphere in the troposphere is not parallel to the dry adiabats (Fig. 18.2), but crosses the adiabats toward warmer potential temperatures as altitude increases.

That same standard atmosphere is replotted as the thick dotted grey line in Fig. 18.3, but now in terms of its potential temperature (θ) versus height (z). The standard atmosphere slopes toward warmer potential temperatures at greater altitudes. Such a slope indicates statically stable air; namely, air that opposes vertical motion.

The ABL is often turbulent. Because turbulence causes mixing, the bottom part of the standard atmosphere becomes homogenized. Namely, within the turbulent region, warmer potential-temperature air from the standard atmosphere in the top of the ABL is mixed with cooler potential-temperature air from near the bottom. The resulting mixture has a

INFO • Engineering Boundary Layers

In wind tunnel experiments, the layer of air that turbulently "feels" frictional drag against the bottom wall grows in depth indefinitely (Fig. 18.a). This engineering boundary-layer thickness h grows proportional to the square root of downstream distance x, until hitting the top of the wind tunnel.

On an idealized rotating planet, the Earth's rotation imposes a dynamical constraint on ABL depth (Fig. 18.b). This maximum depth is proportional to the ratio of wind drag (related to the friction velocity u_*, which is a concept discussed later in this chapter) to Earth's rotation (related to the Coriolis parameter f_c, as discussed in the Atmos. Forces & Winds chapter). This dynamic constraint supersedes the turbulence constraint.

For the real ABL on Earth, the strong capping inversion at height z_i makes the ABL unique (Fig. 18.c) compared to other fluid flows. It constrains the ABL thickness and the eddies within it to a maximum size of order 200 m to 4 km. This stratification (thermodynamic) constraint supersedes the others. It means that the temperature structure is always very important for the ABL.

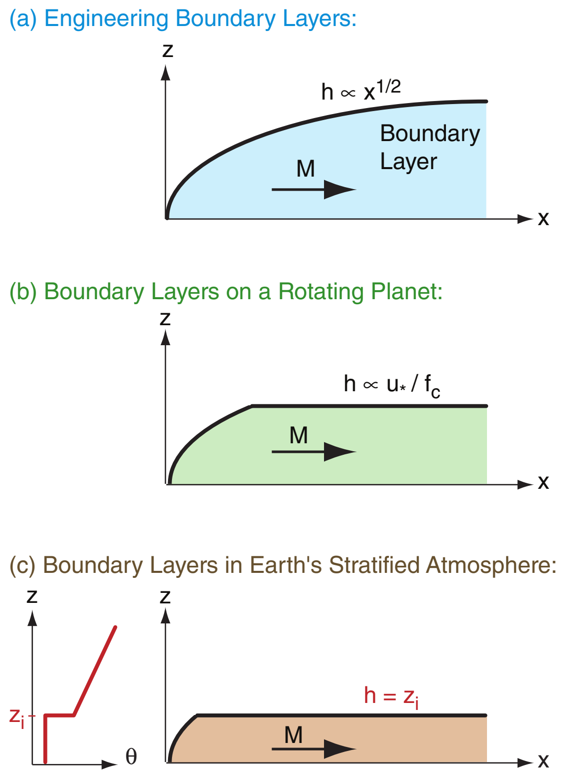


Figure 18.a-c
Comparison of constraints on boundary-layer thickness, h. M is mean wind speed away from the bottom boundary, θ is potential temperature, z is height, and x is downwind distance.

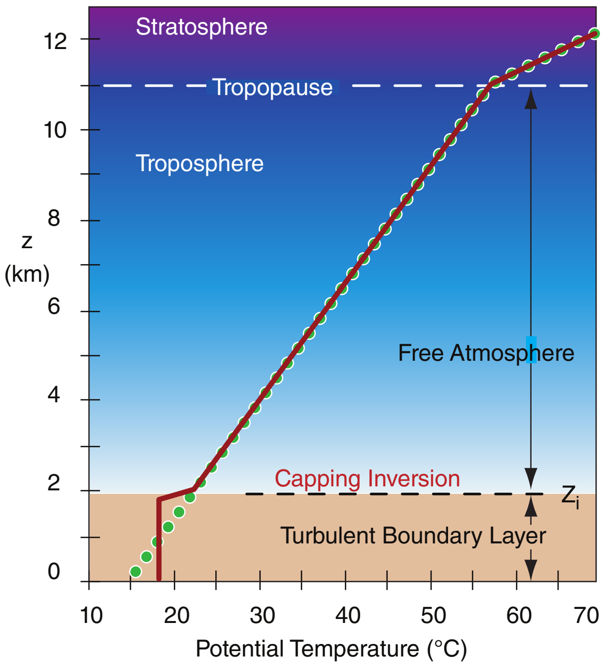


Figure 18.3
Restriction of ABL depth by tropospheric temperature structure during fair weather. The standard atmosphere is the green dotted line. The thick red line shows an idealized temperature profile after the turbulent boundary layer modifies the bottom part of the standard atmosphere.

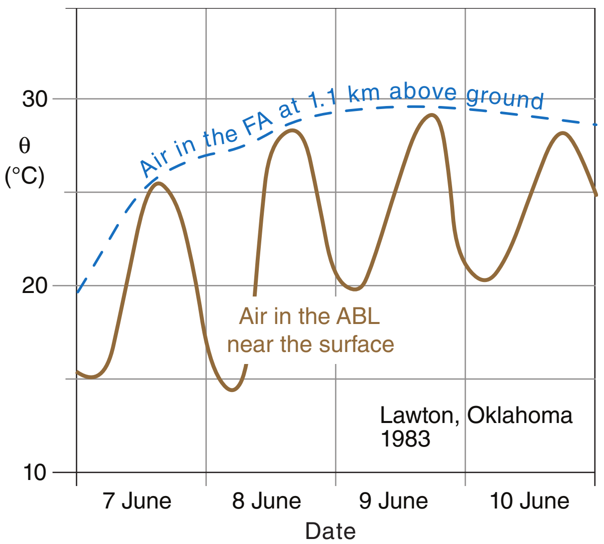


Figure 18.4
Observed variations of potential temperature in the ABL (brown solid line) and the free atmosphere (FA) (blue dashed line). The daily heating and cooling cycle that we are so familiar with near the ground does not exist above the ABL.

medium potential temperature that is uniform with height, as plotted by the thick black line in Fig. 18.3. In situations of vigorous turbulence, the ABL is also called the **mixed layer** (**ML**).

Above the mixed layer, the air is usually un-modified by turbulence, and retains the same temperature profile as the standard atmosphere in this idealized scenario. This tropospheric air above the ABL is known as the **free atmosphere** (**FA**).

As a result of a turbulent mixed layer being adjacent to the unmixed free atmosphere, there is a sharp temperature increase at the mixed layer top. This transition zone is very stable, and is often a **temperature inversion**. Namely, it is a region where temperature increases with height. The altitude of the middle of this inversion is given the symbol z_i, and is a measure of the depth of the turbulent ABL.

The temperature inversion acts like a lid or cap to motions in the ABL. Picture an air parcel from the mixed layer in Fig. 18.3. If turbulence were to try to push it out of the top of the mixed layer into the free atmosphere, it would be so much colder than the surrounding environment that a strong buoyant force would push it back down into the mixed layer. Hence, air parcels, turbulence, and any air pollution in the parcels, are trapped within the mixed layer.

During fair weather, there is always a strong stable layer or temperature inversion capping the ABL. As we have seen, turbulent mixing in the bottom of the statically-stable troposphere creates this cap, and in turn this cap traps turbulence below it.

The capping inversion breaks the troposphere into two parts. Vigorous turbulence within the ABL causes the ABL to respond quickly to surface influences such as heating and frictional drag. However, the remainder of the troposphere does not experience this strong turbulent coupling with the surface, and hence does not experience frictional drag nor a daily heating cycle. Fig. 18.4 illustrates this.

In summary, the bottom 200 m to 4 km of the troposphere is called the atmospheric boundary layer. ABL depth is variable with location and time. Turbulent transport causes the ABL to feel the direct effects of the Earth's surface. The ABL exhibits strong diurnal (daily) variations of temperature, moisture, winds, pollutants, turbulence, and depth in response to daytime solar heating and nighttime IR cooling of the ground. The name "boundary layer" comes from the fact that the Earth's surface is a boundary on the atmosphere, and the ABL is the part of the atmosphere that "feels" this boundary during fair weather.

18.2.2. Synoptic Forcings

Weather patterns such as high (H) and low (L) pressure systems that are drawn on weather maps are known as **synoptic** weather. These large-diameter (≥ 2000 km) systems modulate the ABL. In the N. Hemisphere, ABL winds circulate clockwise and spiral out from high-pressure centers, but circulate counterclockwise and spiral in toward lows (Fig. 18.6). See the Dynamics chapter for details on winds.

The outward spiral of winds around highs is called **divergence**, and removes ABL air horizontally from the center of highs. Conservation of air mass requires **subsidence** (downward moving air) over highs to replace the horizontally diverging air (Fig. 18.5). Although this subsidence pushes free atmosphere air downward, it cannot penetrate into the ABL because of the strong capping inversion. Instead, the capping inversion is pushed downward closer to the ground as the ABL becomes thinner. This situation traps air pollutants in a shallow ABL, causing air stagnation and air-pollution episodes.

Conversely, horizontally converging ABL air around lows is associated with upward motion (Fig. 18.5). Often the synoptic forcings and storms associated with lows are so powerful that they easily lift the capping inversion or eliminate it altogether. This allows ABL air to be deeply mixed over the whole depth of the troposphere by thunderstorms and other clouds. Air pollution is usually reduced during this situation as it is diluted with cleaner air from aloft or is washed out by rain.

Because winds in high-pressure regions are relatively light, ABL air lingers over the surface for sufficient time to take on characteristics of that surface. These characteristics include temperature, humidity, pollution, odor, and others. Such ABL air is called an **airmass**, as discussed in the chapter on Airmasses and Fronts. When two different ABL airmasses leave their genesis regions and are drawn toward each other by a low center, the zone separating those two airmasses is called a **front**.

At a frontal zone, the colder, heavier airmass acts like a wedge under the warm airmass. As winds blow the cold and warm air masses toward each other, the cold wedge causes the warm ABL to peel away from the ground, causing it to ride up over the colder air (Figs. 18.7a & b). Also, thunderstorms can vent ABL air away from the ground (Figs. 18.7a & b). It is mainly in these stormy conditions (statically stable conditions at fronts, and statically unstable conditions at thunderstorms) that ABL air is forced away from the surface.

Although an ABL forms in the advancing airmass behind the front, the warm humid air that was pushed aloft is not called an ABL because it has lost contact with the surface. Instead, this rising warm air cools, allowing water vapor to condense and make the clouds that we often associate with fronts.

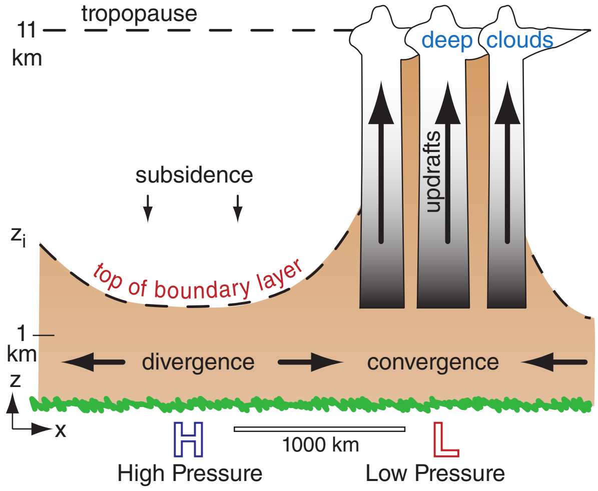


Figure 18.5
Influence of synoptic-scale vertical circulations on the ABL.

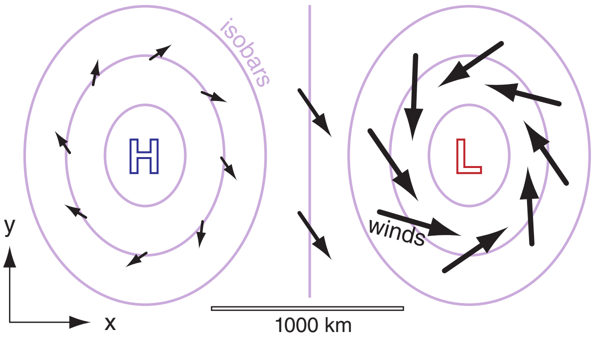


Figure 18.6
Synoptic-scale horizontal winds (arrows) in the ABL near the surface. Thin lavender-colored lines are isobars around surface high (H) and low (L) pressure centers.

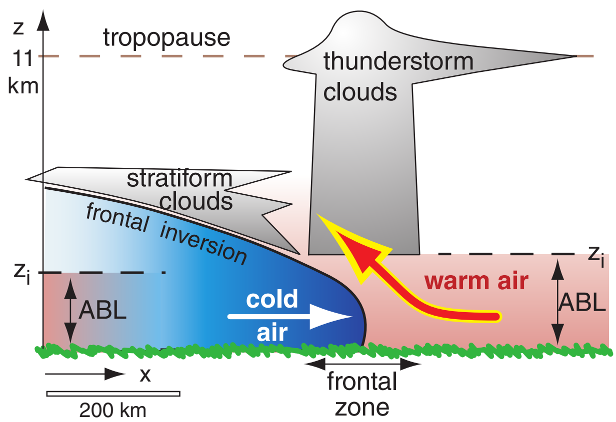


Figure 18.7a
Idealized ABL modification near a frontal zone.

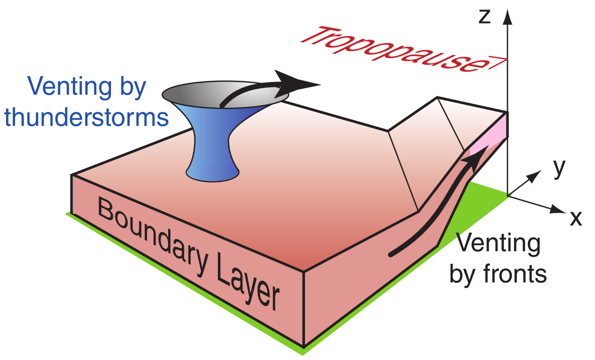


Figure 18.7b
Idealized venting of ABL air away from the surface during stormy weather.

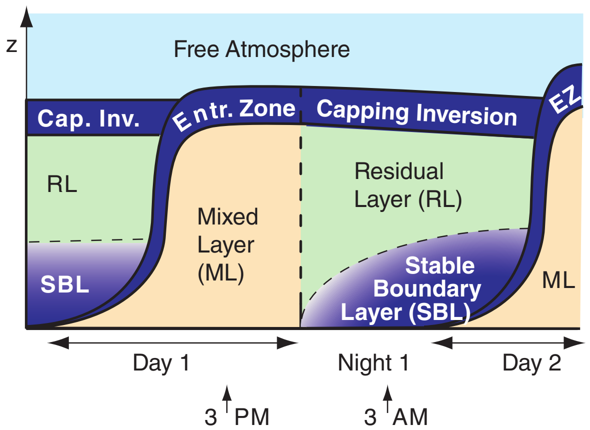


Figure 18.8
Components of the boundary layer during fair weather in summer over land. Tan indicates nonlocally statically unstable air, light green (as in the RL) is neutral stability, and darker blues indicate stronger static stability.

For synoptic-scale low-pressure systems, it is difficult to define a separate ABL, so boundary-layer meteorologists study the air below cloud base. The remainder of this chapter focuses on fair-weather ABLs associated with high-pressure systems.

18.3. ABL STRUCTURE AND EVOLUTION

The fair-weather ABL consists of the components sketched in Fig. 18.8. During daytime there is a statically unstable **mixed layer** (**ML**). At night, a statically **stable boundary layer** (**SBL**) forms under a statically neutral **residual layer** (**RL**). The RL contains the pollutants and moisture from the previous day's mixed layer, but is not very turbulent.

The bottom 20 to 200 m of the ABL is called the **surface layer** (SL, Fig. 18.9). Here frictional drag, heat conduction, and evaporation from the surface cause substantial variations of wind speed, temperature, and humidity with height. However, turbulent fluxes are relatively uniform with height; hence, the surface layer is known as the **constant flux layer**.

Separating the **free atmosphere** (FA) from the mixed layer is a strongly stable **entrainment zone** (**EZ**) of intermittent turbulence. Mixed-layer depth z_i is the distance between the ground and the middle of the EZ. At night, turbulence in the EZ ceases, leaving a non-turbulent layer called the **capping inversion** (**CI**) that is still strongly statically stable.

Typical vertical profiles of temperature, potential temperature, humidity (mixing ratio), and wind speed are sketched in Fig. 18.9. The "day" portion (Fig. 18.9a) corresponds to the 3 PM time indicated in Fig. 18.8, while "night" (Fig. 18.9b) is for 3 AM.

Next, we will look at ABL temperature, winds, and turbulence in more detail.

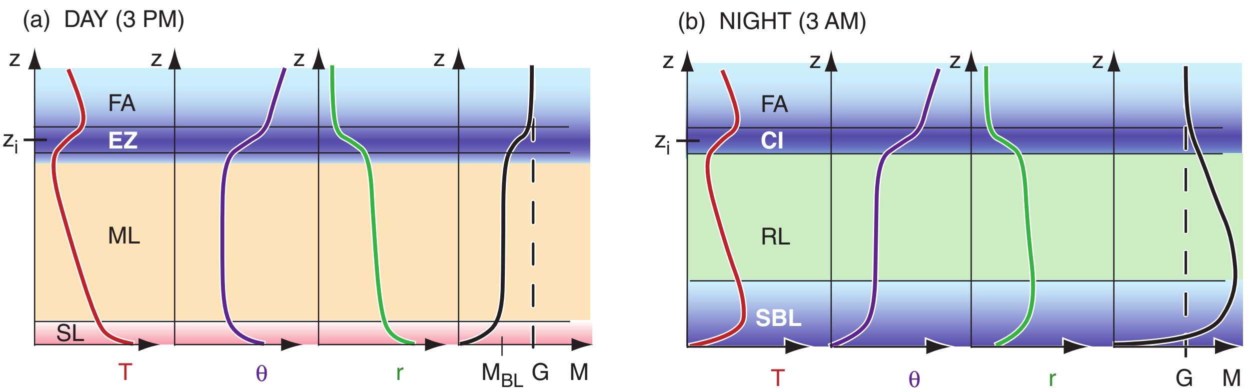


Figure 18.9
Typical vertical profiles of temperature (T), potential temperature (θ), mixing ratio (r, see the Water Vapor chapter), and wind speed (M) in the ABL. The dashed line labeled G is the geostrophic wind speed (a theoretical wind in the absence of surface drag, see the Atmos. Forces & Winds chapter). M_{BL} is average wind speed in the ABL. Shading corresponds to shading in Fig. 18.8: tan is statically unstable, green is neutral, blue is statically stable, and darker blues are more stable.

18.4. TEMPERATURE

The capping inversion traps in the ABL any heating and water evaporation from the surface. As a result, heat accumulates within the ABL during daytime, or whenever the surface is warmer than the air. Cooling (actually, heat loss) accumulates during nighttime, or whenever the surface is colder than the air. Thus, the temperature structure of the ABL depends on the accumulated heating or cooling.

18.4.1. Cumulative Heating or Cooling

The cumulative effect of surface heating and cooling on ABL evolution is more important than the instantaneous heat flux. This cumulative heating or cooling Q_A equals the area under the curve of heat flux vs. time (Fig. 18.10). We will examine cumulative nighttime cooling separately from cumulative daytime heating.

18.4.1.1. Nighttime

During clear nights over land, heat flux from the air to the cold surface is relatively constant with time (dark shaded portion of Fig. 18.10) due to persistent infrared radiation from the ground to space. If we define t as the time since cooling began, then the accumulated cooling per unit surface area is:

$$Q_A = \mathbb{F}_{H\ night} \cdot t \tag{18.1a}$$

For night, Q_A is a negative number because $\mathbb{F}_H$ is negative for cooling. Q_A has units of J m^{-2}.

Dividing eq. (18.1a) by air density and specific heat ($\rho_{air} \cdot C_p$) gives the kinematic form

$$Q_{Ak} = F_{H\ night} \cdot t \tag{18.1b}$$

where Q_{Ak} has units of K·m.

For a night with variable cloudiness that causes a variable surface heat flux (see the Solar & IR Radiation chapter), use the average value of $\mathbb{F}_H$ or F_H.

18.4.1.2. Daytime

On clear days, the nearly sinusoidal variation of solar elevation and downwelling solar radiation (Fig. 2.13 in the Solar & IR Radiation chapter) causes nearly sinusoidal variation of surface net heat flux (Fig. 3.9 in the Heat chapter). Let t be the time since $\mathbb{F}_H$ becomes positive in the morning, D be the total duration of positive heat flux, and $\mathbb{F}_{H\ max}$ be the peak value of heat flux (Fig. 18.10). These values can be found from data such as in Fig. 3.9. The accumulated daytime heating per unit surface area (units of J m^{-2}) is:

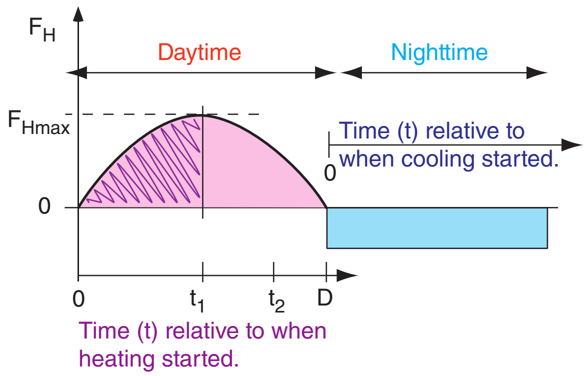


Figure 18.10
Idealization of the heat flux curve over land during fair weather, from Fig. 3.9 of the Heat chapter. Pink shading shows total accumulated heating during the day, while the magenta hatched region shows the portion of heating accumulated up to time t_1 after heating started. Blue shading shows total accumulated cooling during the night.

HIGHER MATH • Cumulative Heating

Derivation of Daytime Cumulative Heating

Assume that the kinematic heat flux is approximately sinusoidal with time:

$$F_H = F_{H\max} \cdot \sin(\pi \cdot t / D)$$

with t and D defined as in Fig. 18.10 for daytime. Integrating from time $t = 0$ to arbitrary time t :

$$Q_{Ak} = \int_{t'=0}^{t} F_H dt' = F_{H\max} \cdot \int_{t'=0}^{t} \sin(\pi \cdot t' / D) dt'$$

where t' is a dummy variable of integration.

From a table of integrals, we find that:

$$\int \sin(a \cdot x) = -(1/a) \cdot \cos(a \cdot x)$$

Thus, the previous equation integrates to:

$$Q_{Ak} = \frac{-F_{H\max} \cdot D}{\pi} \cdot \cos\left(\frac{\pi \cdot t}{D}\right)\Bigg|_0^t$$

Plugging in the two limits gives:

$$Q_{Ak} = \frac{-F_{H\max} \cdot D}{\pi} \cdot \left[\cos\left(\frac{\pi \cdot t}{D}\right) - \cos(0)\right]$$

But the cos(0) = 1, giving the final answer (eq. 18.2b):

$$Q_{Ak} = \frac{F_{H\max} \cdot D}{\pi} \cdot \left[1 - \cos\left(\frac{\pi \cdot t}{D}\right)\right]$$

Sample Application

Use Fig. 3.9 from the Heat chapter to estimate the accumulated heating and cooling in kinematic form over the whole day and whole night.

Find the Answer

By eye from Fig. 3.9:

Day: $\mathbb{F}_{H\,max} \approx$ 150 W·m^{-2} , D = 8 h

Night: $\mathbb{F}_{H\,night} \approx$ –50 W·m^{-2} (averaged)

Given: Day: $t = D$ = 8 h = 28800. s

Night: t = 24h – D = 16 h = 57600. s

Find: Q_{Ak} = ? K·m for day and for night

First convert fluxes to kinematic form by dividing by $\rho_{air}\cdot C_p$:

$F_{H\,max} = \mathbb{F}_{H\,max} / \rho_{air}\cdot C_p$

= (150 W·m^{-2}) / [1231(W·m^{-2})/(K·m s^{-1})]

= 0.122 K·m s^{-1}

$F_{H\,night}$ = (–50W·m^{-2})/[1231(W·m^{-2})/(K·m s^{-1})]

= –0.041 K·m s^{-1}

Day: use eq. (18.2b):

$$Q_{Ak} = \frac{(0.122\ \text{K}\cdot\text{m/s})\cdot(28800\ \text{s})}{3.14159}\cdot[1-\cos(\pi)]$$

= **2237 K·m**

Night: use eq. (18.1b):

Q_{Ak} = (–0.041 K·m s^{-1})·(57600. s) = **–2362 K·m**

Check: Units OK. Physics OK.

Exposition: The heating and cooling are nearly equal for this example, implying that the daily average temperature is fairly steady.

$$Q_A = \frac{\mathbb{F}_{H\,max}\cdot D}{\pi}\cdot\left[1-\cos\left(\frac{\pi\cdot t}{D}\right)\right] \qquad (18.2a)$$

In kinematic form (units of K·m), this equation is

$$Q_{Ak} = \frac{F_{H\,max}\cdot D}{\pi}\cdot\left[1-\cos\left(\frac{\pi\cdot t}{D}\right)\right] \qquad (18.2b)$$

18.4.2. Temperature-Profile Evolution

18.4.2.1. Idealized Evolution

A typical afternoon temperature profile is plotted in Fig. 18.11a. During the daytime, the environmental lapse rate in the mixed layer is nearly adiabatic. The unstable surface layer (plotted but not labeled in Fig. 18.11) is in the bottom part of the mixed layer. Warm blobs of air called **thermals** rise from this surface layer up through the mixed layer, until they hit the temperature inversion in the entrainment zone. Fig. 18.12a shows a closer view of the surface layer (bottom 5 - 10% of the ABL).

These thermal circulations create strong turbulence and cause pollutants, potential temperature, and moisture to be well mixed in the vertical (hence the name **mixed layer**). The whole mixed layer, surface layer, and bottom portion of the entrainment zone are statically unstable.

In the entrainment zone, free-atmosphere air is incorporated or **entrained** into the mixed layer, causing the mixed-layer depth to increase during the day. Pollutants trapped in the mixed layer cannot escape through the EZ, although cleaner, drier free atmosphere air is entrained into the mixed layer. Thus, the EZ is a one-way valve.

At night, the bottom of the mixed layer becomes chilled by contact with the radiatively-cooled

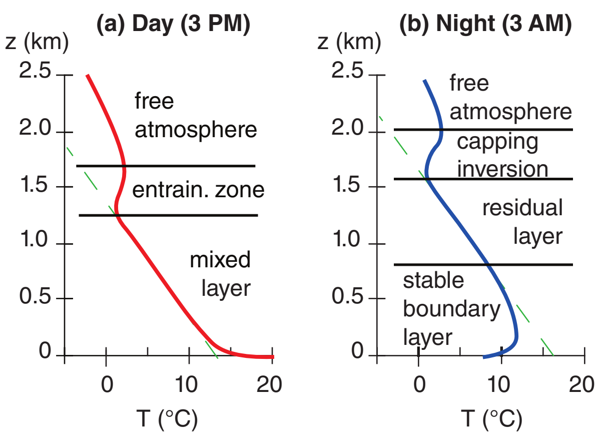


Figure 18.11
Examples of boundary-layer temperature profiles during (a) day and (b) night during fair weather over land. The dashed green line indicates the diabatic lapse rate. The heights shown here are illustrative only. In the real ABL the heights can be greater or smaller, depending on location, time, and season.

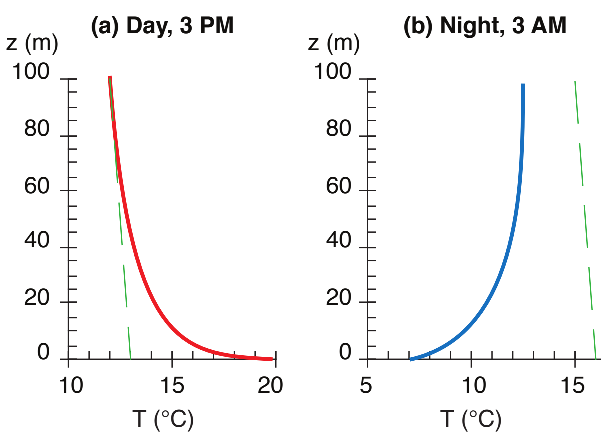


Figure 18.12
Examples of surface-layer temperature profiles during (a) day & (b) night. The dashed green line shows the adiabatic lapse rate. Again, heights are illustrative only. Actual heights might differ.

ground. The result is a stable ABL. The bottom portion of this stable ABL is the surface layer (again not labeled in Fig. 18.11b, but sketched in Fig. 18.12b).

Above the stable ABL is the residual layer. It has not felt the cooling from the ground, and hence retains the adiabatic lapse rate from the mixed layer of the previous day. Above that is the capping temperature inversion, which is the non-turbulent remnant of the entrainment zone.

18.4.2.2. Seasonal Differences

During summer at mid- and high-latitudes, days are longer than nights, allowing more heating to occur during day than cooling at night for fair weather over land. This causes net heating over a 24-h period, as illustrated in Figs. 18.13 a & b. The convective mixed layer starts shallow in the morning, but rapidly grows through the residual layer. In the afternoon, it continues to rise slowly into the free atmosphere. If the air contains sufficient moisture, cumuliform clouds can exist. At night, cooling creates a shallow stable ABL near the ground, but leaves a thick residual layer above it.

During winter at mid- and high-latitudes, more cooling occurs during the long nights than heating during the short days, in fair weather. Stable ABLs dominate over land, and there is net temperature decrease over 24 hours (Figs. 18.13 c & d). Any non-frontal clouds present are typically stratiform or fog. Any residual layer that forms early in the night is quickly consumed by the growing stable ABL.

Fig. 18.14 shows the corresponding structure of the ABL. Although both the mixed layer and residual layer have nearly adiabatic temperature profiles, the mixed layer is nonlocally unstable, while the residual layer is neutral. This difference causes pollutants to disperse at different rates in those two regions.

If the wind moves ABL air over surfaces of different temperatures, then ABL structures can evolve in space, rather than in time. For example, suppose the numbers along the abscissa in Fig. 18.14b repre-

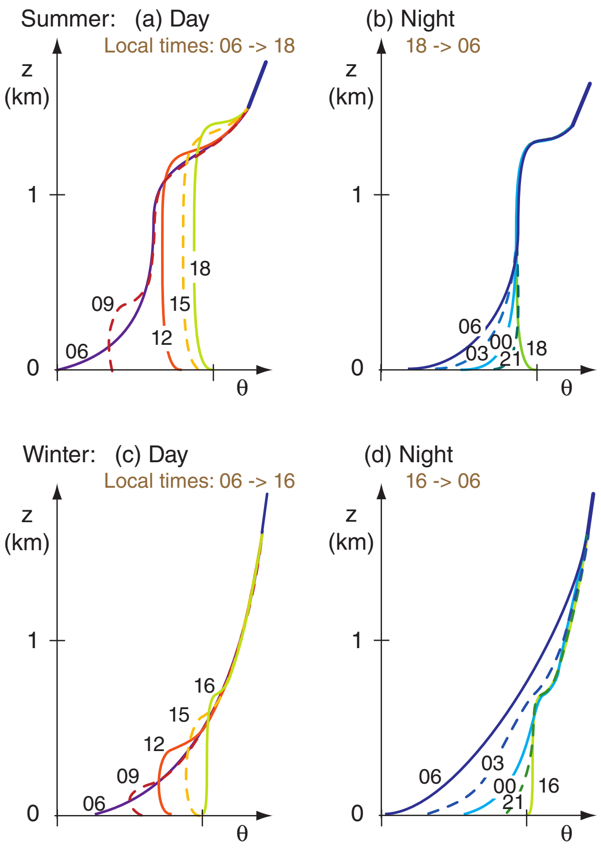


Figure 18.13
Evolution of potential temperature θ profiles for fair weather over land. Curves are labeled with local time in hours. The summer nighttime curves begin at 18 h local time, which roughly corresponds to the final sounding from the previous day. Winter has shorter daylight hours, so the evening transition occurs at 16 h local time, in this midlatitude idealization.

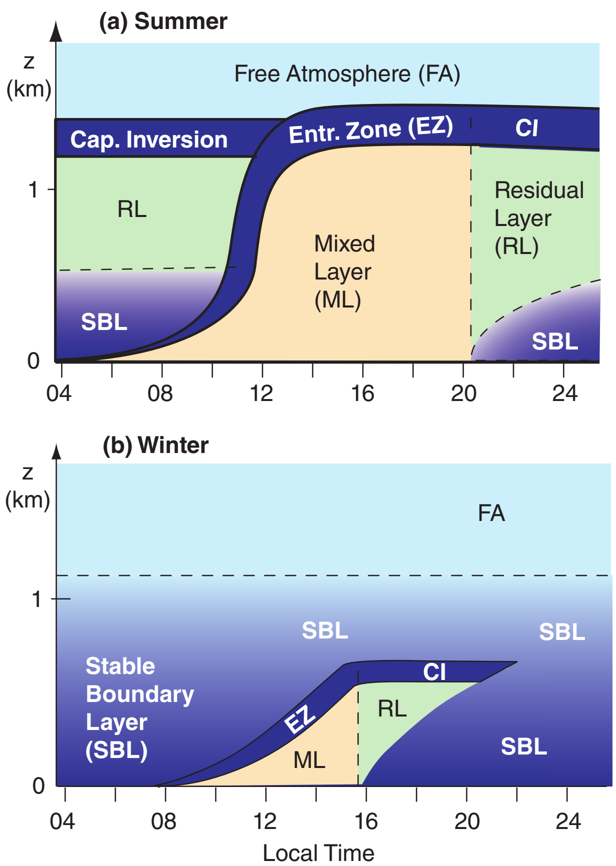


Figure 18.14
Daily evolution of boundary-layer structure for summer and winter, for fair weather over land. CI = Capping Inversion. Shading indicates static stability: tan = unstable, green = neutral (as in the RL), darker blues = stronger static stabilities.

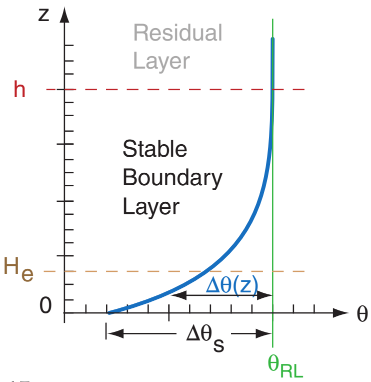
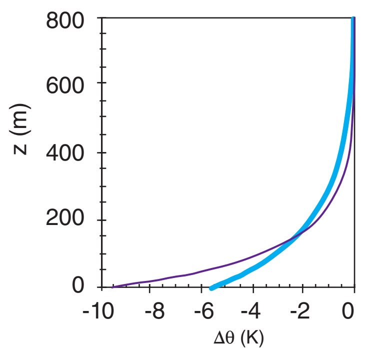


Figure 18.15
Idealized exponential-shaped potential temperature profile in the stable (nighttime) boundary layer.

> **Sample Application** (§)
> Estimate the potential temperature profile at the end of a 12-hour night for two cases: windy (10 m s^{-1}) and less windy (5 m s^{-1}). Assume Q_{Ak} = –1000 K·m.
>
> **Find the Answer**
> Given: Q_{Ak} = –1000 K·m, t = 12 h
> (a) M_{RL} = 10 m s^{-1}, (b) M_{RL} = 5 m s^{-1}
> Find: θ vs. z
>
> Assume: flat prairie. To plot profile, need H_e & $\Delta\theta_s$.
> Use eq. (18.4):
>
> $$H_e \approx (0.15\text{m}^{1/4}\cdot\text{s}^{1/4})\cdot(10\text{m}\cdot\text{s}^{-1})^{3/4}\cdot(43200\text{s})^{1/2}$$
> (a) H_e = 175 m
> $$H_e \approx (0.15\text{m}^{1/4}\cdot\text{s}^{1/4})\cdot(5\text{m}\cdot\text{s}^{-1})^{3/4}\cdot(43200\text{s})^{1/2}$$
> (b) H_e = 104 m
>
> Use eq. (18.5):
> (a) $\Delta\theta_s$ = (–1000K·m)/(175) = –5.71 °C
> (b) $\Delta\theta_s$ = (–1000K·m)/(104) = –9.62 °C
> Use eq. (18.3) in a spreadsheet to compute the potential temperature profiles, using H_e and $\Delta\theta_s$ from above:
>
> 
>
>
> **Check**: Units OK, Physics OK. Graph OK.
> **Exposition**: The windy case (thick cyan line) is not as cold near the ground, but the cooling extends over a greater depth than for the less-windy case (thin line).

sent distance x (km), with wind blowing from left to right over a lake spanning 8 ≤ x ≤ 16 km. The ABL structure in Fig. 18.14b could occur at midnight in mid-latitude winter for air blowing over snow-covered ground, except for the unfrozen lake in the center. If the lake is warmer than the air, it will create a mixed layer that grows as the air advects across the lake.

Don't be lulled into thinking the ABL evolves the same way at every location or at similar times. The most important factor is the temperature difference between the surface and the air. If the surface is warmer, a mixed layer will develop regardless of the time of day. Similarly, colder surfaces will create stable ABLs.

18.4.3. Stable ABL Temperature

Stable ABLs are quite complex. Turbulence can be intermittent, and coupling of air to the ground can be quite weak. In addition, any slope of the ground causes the cold air to drain downhill. Cold, downslope winds are called **katabatic winds**, as discussed in the Regional Winds chapter.

For a simplified case of a turbulent, stable ABL over a flat surface during light winds, the potential temperature profile is approximately exponential with height (Fig. 18.15):

$$\Delta\theta(z) = \Delta\theta_s \cdot e^{-z/H_e} \qquad (18.3)$$

where $\Delta\theta(z) = \theta(z) - \theta_{RL}$ is the potential temperature difference between the air at height z and the air in the residual layer. $\Delta\theta(z)$ is negative.

The value of this difference near the ground is defined to be $\Delta\theta_s = \Delta\theta(z{=}0)$, and is sometimes called the **strength** of the stable ABL. H_e is an **e-folding height** for the exponential curve. The actual **depth** h of the stable ABL is roughly $h = 5\cdot H_e$.

Depth and strength of the stable ABL grow as the cumulative cooling Q_{Ak} increases with time:

$$H_e \approx a \cdot M_{RL}^{\;3/4} \cdot t^{1/2} \qquad (18.4)$$

$$\Delta\theta_s = \frac{Q_{Ak}}{H_e} \qquad (18.5)$$

where a = 0.15 m$^{1/4}$·s$^{1/4}$ for flow over a flat prairie, and where M_{RL} is the wind speed in the residual layer. Because the cumulative cooling is proportional to time, both the depth $5\cdot H_e$ and strength $\Delta\theta_s$ of the stable ABL increase with the square root of time. Thus, the fast growth rate of the SBL early in the evening decreases to a much slower growth rate by the end of the night.

18.4.4. Mixed-Layer (ML) Temperature

18.4.4.1. Evolution

The shape of the potential temperature profile in the mixed layer is simple. To good approximation it is uniform with height. Of more interest is the evolution of mixed-layer average θ and z_i with time.

Use the potential temperature profile at the end of the night (early morning) as the starting sounding for forecasting daytime temperature profiles. In real atmospheres, the sounding might not be a smooth exponential as idealized in the previous subsection. The method below works for arbitrary shapes of the initial potential temperature profile.

A graphical solution is easiest. First, plot the early-morning sounding of θ vs. z. Next, determine the cumulative daytime heating Q_{Ak} that occurs between sunrise and some time of interest t_1, using eq. (18.2b). This heat warms the air in the ABL; thus the area under the sounding equals the accumulated heating (and also has units of K·m). Plot a vertical line of constant θ between the ground and the sounding, so that the area under the curve (hatched in Fig. 18.16a) equals the cumulative heating.

This vertical line gives the potential temperature of the mixed layer $\theta_{ML}(t_1)$. The height where this vertical line intersects the early-morning sounding defines the mixed-layer depth $z_i(t_1)$. As cumulative heating increases with time during the day ($Area_2$ = total pink-shaded region at time t_2), the mixed layer becomes warmer and deeper (Fig. 18.16a). The resulting potential temperature profiles during the day are sketched in Fig. 18.16b. This method of finding mixed layer growth is called the **encroachment method**, or **thermodynamic method**, and explains roughly 90% of typical mixed-layer growth on sunny days with winds less than 10 m s^{-1}.

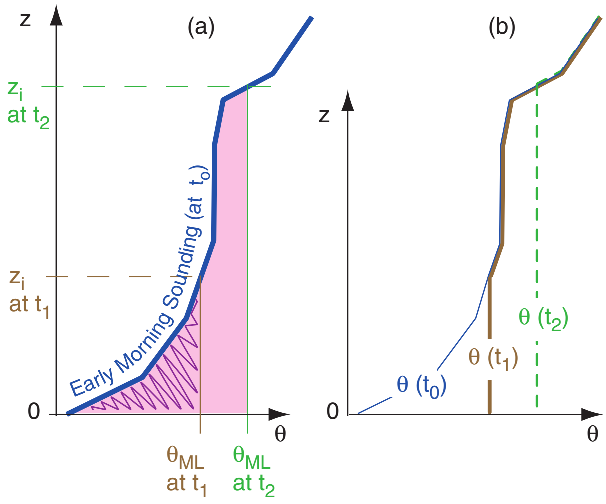


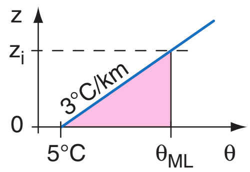
Figure 18.16
Evolution of the mixed layer with time, as cumulative heating increases the areas (hatched, then pink) under the curves.

Sample Application

Given an early morning sounding with surface temperature 5°C and lapse rate $\Delta\theta/\Delta z$ = 3 K km^{-1}. Find the mixed-layer potential temperature and depth at 10 AM, when the cumulative heating is 500 K·m.

Find the Answer

Given: θ_{sfc} = 5°C, $\Delta\theta/\Delta z$ = 3 K km^{-1},
Q_{Ak} =0.50 K·km
Find: θ_{ML} = ? °C, z_i = ? km
Sketch:



The area under this simple sounding is the area of a triangle: $Area = 0.5\cdot[base]\cdot(height)$,

$Area = 0.5\cdot[(\Delta\theta/\Delta z)\cdot z_i]\cdot(z_i)$

0.50 K·km = 0.5·(3 K km^{-1})·z_i^2

Rearrange and solve for z_i:

z_i = **0.577 km**

Next, use the sounding to find the θ_{ML}:

$\theta_{ML} = \theta_{sfc} + (\Delta\theta/\Delta z)\cdot z_i$

θ_{ML} = (5°C) + (3°C km^{-1})·(0.577 km) = **6.73 °C**

Check: Units OK. Physics OK. Sketch OK.

Exposition: Arbitrary soundings can be approximated by straight line segments. The area under such a sounding consists of the sum of areas within trapezoids under each line segment. Alternately, draw the early-morning sounding on graph paper, count the number of little grid boxes under the sounding, and multiply by the area of each box.

18.4.4.2. Entrainment

As was mentioned earlier, the turbulent mixed layer grows by entraining non-turbulent air from the free atmosphere. One can idealize the mixed layer as a **slab model** (Fig. 18.17a), with constant potential temperature in the mixed layer, and a jump of potential temperature (Δθ) at the EZ.

Entrained air from the free atmosphere has warmer potential temperature than air in the mixed layer. Because this warm air is entrained downward, it corresponds to a negative heat flux $F_{H\,zi}$ at the top of the mixed layer. The heat-flux profile (Fig. 18.17b) is often linear with height, with the most negative value marking the top of the mixed layer.

The entrainment rate of free atmosphere air into the mixed layer is called the **entrainment velocity**, w_e, and can never be negative. The entrainment velocity is the volume of entrained air per unit horizontal area per unit time. In other words it is a volume flux, which has the same units as velocity.

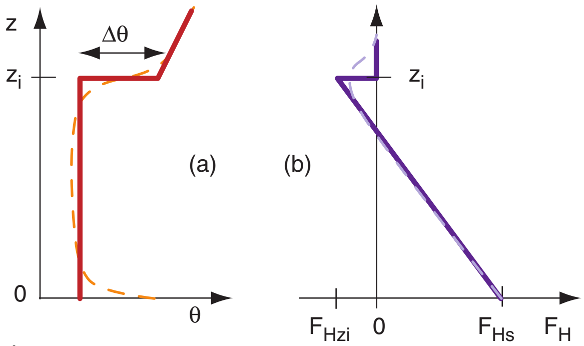


Figure 18.17
(a) Slab idealization of the mixed layer (solid line) approximates a more realistic potential temperature (θ) profile (dashed line). (b) Corresponding real and idealized heat flux (F_H) profiles.

Sample Application
The mixed layer depth increases at the rate of 300 m h^{-1} in a region with weak subsidence of 2 mm s^{-1}. If the inversion strength is 0.5°C, estimate the surface kinematic heat flux.

Find the Answer
Given: w_s= –0.002 m s^{-1}, $\Delta z_i/\Delta t$= 0.083 m s^{-1}, Δθ = 0.5°C
Find: $F_{H\,sfc}$ = ? °C·m s^{-1}

First, solve eq. (18.6) for entrainment velocity:
$w_e = \Delta z_i/\Delta t - w_s$ = [0.083 – (–0.002)] m s^{-1} = 0.085 m s^{-1}

Next, use eq. (18.7):
F_{Hzi} = (–0.085 m s^{-1})·(0.5°C) = –0.0425 °C·m s^{-1}

Finally, rearrange eq. (18.8):
$F_{H\,sfc}= -F_{Hzi}/A$= –(–0.0425 °C·m s^{-1})/0.2 = **0.21°C·m** s^{-1}

Check: Units OK. Magnitude and sign OK.
Exposition: Multiplying by $\rho \cdot C_P$ for the bottom of the atmosphere gives a dynamic heat flux of $\mathbb{F}_H$ = 259 W m^{-2}. This is roughly half of the value of max incoming solar radiation from Fig. 2.13 in the Solar & IR Radiation chapter.

Sample Application
Find the entrainment velocity for a potential temperature jump of 2°C at the EZ, and a surface kinematic heat flux of 0.2 K·m s^{-1}.

Find the Answer
Given: $F_{H\,sfc}$ = 0.2 K·m s^{-1}, Δθ = 2 °C = 2 K
Find: w_e = ? m s^{-1}

Use eq. (18.9):

$$w_e \cong \frac{A \cdot F_{H\,sfc}}{\Delta\theta} = \frac{0.2 \cdot (0.2\,\mathrm{K \cdot m/s})}{(2\,\mathrm{K})} = \mathbf{0.02\ m}\ \mathrm{s}^{-1}$$

Check: Units OK. Physics OK.
Exposition: While 2 cm s^{-1} seems small, when applied over 12 h of daylight works out to z_i = 864 m, which is a reasonable mixed-layer depth.

The rate of growth of the mixed layer during fair weather is

$$\frac{\Delta z_i}{\Delta t} = w_e + w_s \qquad \bullet(18.6)$$

where w_s is the synoptic scale vertical velocity, and is negative for the subsidence that is typical during fair weather (recall Fig. 18.5).

The kinematic heat flux at the top of the mixed layer is

$$F_{Hzi} = -w_e \cdot \Delta\theta \qquad \bullet(18.7)$$

where the sign and magnitude of the temperature jump is defined by Δθ = θ(just above z_i) – θ(just below z_i). Greater entrainment across stronger temperature inversions causes greater heat-flux magnitude. Similar relationships describe entrainment fluxes of moisture, pollutants, and momentum as a function of the jumps in humidity, pollution concentration, and wind speed, respectively, at the ML top. As for temperature, the jump is defined as the value above z_i minus the value below z_i. Entrainment velocity has the same value for all variables.

During free convection (when winds are weak and thermal convection is strong), the entrained kinematic heat flux is approximately 20% of the surface heat flux:

$$F_{H\,zi} \cong -A \cdot F_{H\,sfc} \qquad (18.8)$$

where A = 0.2 is called the **Ball ratio**, and $F_{H\,sfc}$ is the surface kinematic heat flux. This special ratio works only for heat, and does not apply to other variables. During windier conditions, A can be greater than 0.2. Eq. (18.8) was used in the Heat chapter to get the vertical heat-flux divergence (eq. 3.40 & 3.41), a term in the Eulerian heat budget.

Combining the two equations above gives an approximation for the entrainment velocity during free convection:

$$w_e \cong \frac{A \cdot F_{H\,sfc}}{\Delta\theta} \qquad \bullet(18.9)$$

Combining this with eq. (18.6) gives a mixed-layer growth equation called the **flux-ratio method**. From these equations, we see that stronger capping inversions cause slower growth rate of the mixed layer, while greater surface heat flux (e.g., sunny day over land) causes faster growth. The flux-ratio method and the thermodynamic method usually give equivalent results for mixed layer growth during free convection.

For stormy conditions near thunderstorms or fronts, the ABL top is roughly at the tropopause. Alternately, in the Atmos. Forces & Winds chapter (Mass Conservation section) are estimates of z_i for bad weather.

18.5. WIND

For any given weather condition, there is a theoretical equilibrium wind speed, called the **geostrophic wind** G, that can be calculated for frictionless conditions (see the Atmos. Forces & Winds chapter). However, steady-state winds in the ABL are usually slower than geostrophic (i.e., **subgeostrophic**) because of frictional and turbulent drag of the air against the surface, as was illustrated in Fig. 18.9a.

Turbulence continuously mixes slower air from close to the ground with faster air from the rest of the ABL, causing the whole ABL to experience drag against the surface and to be subgeostrophic. This vertically averaged steady-state ABL wind M_{BL} is derived in the Atmos. Forces & Winds chapter. The actual wind speed over a large central region of the ABL is nearly equal to the theoretical M_{BL} speed.

Winds closer to the surface (in the **surface layer, SL**) are even slower (Fig. 18.9a). Wind-profile shapes in the SL are empirically found to be similar to each other when scaled with appropriate length and velocity scales. This approach, called **similarity theory**, is described later in this section.

18.5.1. Wind Profile Evolution

Over land during fair weather, the winds often experience a diurnal cycle as illustrated in Fig. 18.18. For example, a few hours after sunrise, say at 9 AM local time, there is often a shallow mixed layer, which is 300 m thick in this example. Within this shallow mixed layer the ABL winds are uniform with height, except near the surface where winds approach zero.

As the day progresses, the mixed layer deepens, so by 3 PM a deep layer of subgeostrophic winds fills the ABL. Winds remain moderate near the ground as turbulence mixes down faster winds from higher in the ABL. After sunset, turbulence intensity usually diminishes, allowing surface drag to reduce the winds at ground level. However, without turbulence, the air in the mid-ABL no longer feels drag against the surface, and begins to accelerate.

By 3 AM, the winds a few hundred meters above ground can be supergeostrophic, even though the winds at the surface might be calm. This low-altitude region of supergeostrophic winds is called a **nocturnal jet**. This jet can cause rapid horizontal transport of pollutants, and can feed moisture into thunderstorms. Then, after sunrise, the nocturnal jet disappears as turbulence causes surface drag to increase and as slower air is mixed from below.

For measurements made at fixed heights on a very tall tower, the same wind-speed evolution is

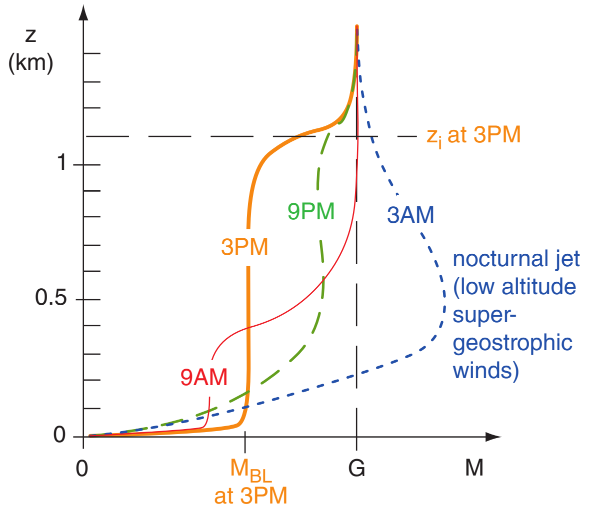


Figure 18.18
Typical ABL wind-profile evolution during fair weather over land. G is geostrophic wind speed. M_{BL} is average ABL wind speed and z_i is the average mixed-layer depth at 3 PM local time. The region of **supergeostrophic** *(faster-than-geostrophic: M > G) winds is called a nocturnal jet.*

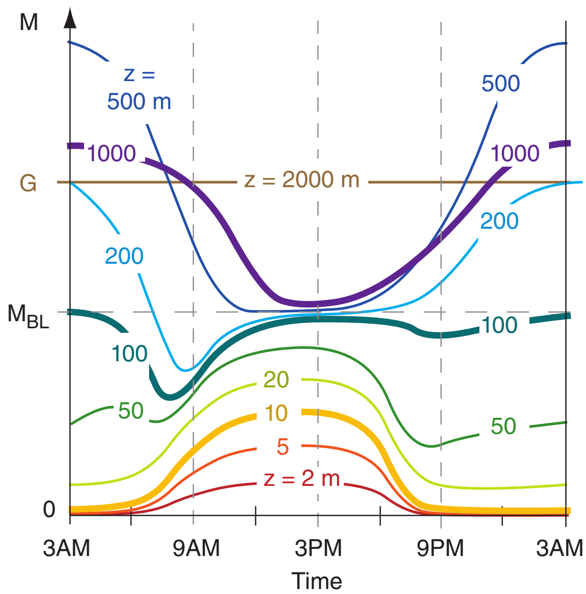


Figure 18.19
Typical ABL wind speed evolution at different heights above ground. G is geostrophic wind speed, M_{BL} is average ABL wind during mid afternoon, and the vertical time lines correspond to the profiles of Fig. 18.18.

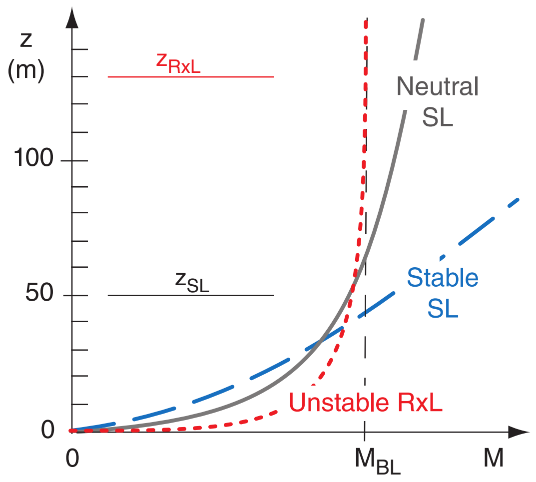


Figure 18.20
Typical wind speed profiles in the surface layer (SL, bottom 5% of the ABL) and radix layer (RxL, bottom 20% of the ABL), for different static stabilities. z_{RxL} *and* z_{SL} *give order-of-magnitude depths for the radix layer and surface layer.*

Table 18-1. The Davenport-Wieringa roughness-length z_o (m) classification, with approximate drag coefficients C_D (dimensionless).

z_o (m)	Classification	C_D	Landscape
0.0002	sea	0.0014	sea, paved areas, snow-covered flat plain, tide flat, smooth desert
0.005	smooth	0.0028	beaches, pack ice, morass, snow-covered fields
0.03	open	0.0047	grass prairie or farm fields, tundra, airports, heather
0.1	roughly open	0.0075	cultivated area with low crops & occasional obstacles (single bushes)
0.25	rough	0.012	high crops, crops of varied height, scattered obstacles such as trees or hedgerows, vineyards
0.5	very rough	0.018	mixed farm fields and forest clumps, orchards, scattered buildings
1.0	closed	0.030	regular coverage with large-sized obstacles with open spaces roughly equal to obstacle heights, suburban houses, villages, mature forests
≥ 2	chaotic	≥0.062	centers of large towns and cities, irregular forests with scattered clearings

shown in Fig. 18.19. Below 20 m altitude, winds are often calmer at night, and increase in speed during daytime. The converse is true above 1000 m altitude, where winds are reduced during the day because of turbulent mixing with slower near-surface air, but become faster at night when turbulence decays.

At the ABL bottom, near-surface wind speed profiles have been found **empirically** (via observations). In the bottom 5% of the statically neutral ABL is the **surface layer**, where wind speeds increase roughly logarithmically with height (Fig. 18.20).

For the statically stable surface layer, this logarithmic profile changes to a more linear form (Fig. 18.20). Winds close to the ground become slower than logarithmic, or near calm. Winds just above the surface layer are often not in steady state, and can temporarily increase to faster than geostrophic (**supergeostrophic**) in a process called an **inertial oscillation** (Figs. 18.9b and 18.20).

The bottom 20% of the convective (unstable) ABL is called the **radix layer** (RxL). Winds in the RxL have an exponential power-law relationship with height. The RxL has faster winds near the surface, but slower winds aloft than the neutral logarithmic profile. After a discussion of drag at the ground, these three wind cases at the bottom of the ABL will be described in more detail.

18.5.2. Drag, Stress, Friction Velocity & Roughness Length

The frictional force between two objects such as the air and the ground is called **drag**. One way to quantify drag is by measuring the force required to push the object along another surface. For example, if you place your textbook on a flat desk, after you first start it moving you must continue to push it with a certain force (i.e., equal and opposite to the drag force) to keep it moving. If you stop pushing, the book stops moving.

Your book contacts the desk with a certain surface area. Generally, larger contact area requires greater force to overcome friction. The amount of friction force per unit surface contact area is called **stress**, τ, and acts parallel to the surface. Contrast this with pressure, which is defined as a force per unit area that is perpendicular to the surface. Units of stress are N m^{-2} (see Appendix A), and could also be expressed as Pascals (Pa) or kiloPascals (kPa).

Stress is felt by both objects that are sliding against each other. For example, if you stack two books on top of each other, then there is friction between both books, as well as between the bottom book and the table. In order to push the bottom book in one direction without moving the top book relative to the table, you must apply a force to the top book in the opposite direction as the bottom book.

Think of air within the ABL as a stack of layers of air, much like a stack of books. Each layer feels stress from the layers above and below it. The bottom layer feels stress against the ground, as well as from the layer of air above. In turn, the surface feels a stress due to air drag. Over the ocean, this **wind stress** drives the **ocean currents**.

In the atmosphere, stress caused by turbulent motions is many orders of magnitude greater than stress caused by molecular viscosity (see the INFO box on this page). For that reason, we often speak of **turbulent stress** instead of frictional stress, and **turbulent drag** rather than frictional drag. This turbulent stress is also called a **Reynolds stress**, after Osborne Reynolds who related this stress to turbulent gust velocities in the late 1800s.

Because air is a fluid, it is often easier to study the stress τ per unit density ρ of air. This is called the **kinematic stress**. The kinematic stress against the Earth's surface is given the symbol u_*^2, where u_* is called the **friction velocity**:

$$u_*^2 = |\tau / \rho| \qquad \bullet(18.10)$$

Typical values range from $u_* = 0$ during calm winds to $u_* = 1$ m s^{-1} during strong winds. Moderate-wind values are often near $u_* = 0.5$ m s^{-1}.

For fluid flow, turbulent stress is proportional to wind speed squared, and also increases with surface roughness. A dimensionless **drag coefficient** C_D relates the kinematic stress to the wind speed M_{10} at z = 10 m.

$$u_*^2 = C_D \cdot M_{10}^2 \qquad \bullet(18.11)$$

The drag coefficient ranges from C_D = 2x10^{-3} over smooth surfaces to 2x10^{-2} over rough or forested surfaces (Table 18-1). It is similar to the bulk heat-transfer coefficient of the Heat chapter.

The surface roughness is usually quantified as an **aerodynamic roughness length** z_o. Table 18-1 shows typical values of the roughness length for various surfaces. Rougher surfaces such as sparse forests have greater values of roughness length than smoother surfaces such as a frozen lake. Roughness lengths in this table are <u>not</u> equal to the heights of the houses, trees, or other roughness elements.

For statically neutral air flow, there is a relationship between drag coefficient and aerodynamic roughness length:

$$C_D = \frac{k^2}{\ln^2(z_R / z_o)} \qquad (18.12)$$

where k = 0.4 is the **von Kármán constant**, and z_R = 10 m is a reference height defined as the standard

Sample Application

Find the drag coefficient in statically neutral conditions to be used with standard surface winds of 5 m s^{-1}, over (a) villages, and (b) grass prairie. Also, find the friction velocity and surface stress.

Find the Answer

Given: z_R = 10 m for "standard" winds
Find: C_D = ? (dimensionless), u_* = ? m s^{-1}, τ = ? N m^{-2}

Use Table 18-1:
(a) z_o = 1 m for villages. (b) z_o = 0.03 m for prairie

Use eq. (18.12) for drag coefficient:

(a) $C_D = \dfrac{0.4^2}{\ln^2(10\text{m} / 1\text{m})}$ = **<u>0.030</u>** (dimensionless)

(b) $C_D = \dfrac{0.4^2}{\ln^2(10\text{m} / 0.03\text{m})}$ = **<u>0.0047</u>** (dimensionless)

Use eq. (18.11) for friction velocity:
(a) $u_*^2 = C_D \cdot M_{10}^2$ = 0.03·(5m s^{-1})^2 = 0.75 m^2·s^{-2}
Thus u_* = **<u>0.87 m</u>** s^{-1}.

(b) Similarly, u_* = **<u>0.34 m</u>** s^{-1}.

Use eq. (18.10) for surface stress, and assume ρ = 1.2 kg m^{-3}:

(a) $\tau = \rho \cdot u_*^2$ = (1.2 kg m^{-3})·(0.75m^2 s^{-2}) =
τ = 0.9 kg·m^{-1}·s^{-2} = **<u>0.9 Pa</u>** (using Appendix A)

(b) τ = 0.14 kg·m^{-1}·s^{-2} = **<u>0.14 Pa</u>**

Check: Units OK. Physics OK.
Exposition: The drag coefficient, friction velocity, and stress are smaller over smoother surfaces.

In this development we examined the stress for fixed wind speed and roughness. However, in nature, greater roughness & greater surface drag causes slower winds (see the Atmos. Forces & Winds chapter).

INFO • Molecular vs. Turbulent Stress

Suppose that u_* = 0.5 m s^{-1} during statically neutral conditions at sea level. Eq. (18.10) give the turbulent (Reynolds) stress: $\tau_{turb} = \rho \cdot u_*^2$ = (1.225 kg m^{-3})·(0.5 m s^{-1})^2 = **3.06x10^{-1} Pa**.

The wind shear associated with this stress is given by the derivative of eq. (18.14a): $\Delta M/\Delta z = u_*/(k \cdot z)$, where k = 0.4 is the von Kármán constant. At z = 10 m, the shear is $\Delta M/\Delta z$ = (0.5 m·s^{-1})/(0.4·10m) = 0.125 s^{-1}.

Molecular stress is $\tau_{mol.} = \mu \cdot \Delta M/\Delta z$, where the molecular viscosity of air is roughly μ = 1.789x10^{-5} kg m^{-1} s^{-1}. Using this with the shear of the previous paragraph gives $\tau_{mol.}$ = **2.24x10^{-6} Pa**. Thus, molecular stress can be neglected compared to turbulent stress.

Sample Application
If the wind speed is 20 m s^{-1} at 10 m height over an orchard, find the friction velocity.

Find the Answer
Given: M_{10} = 20 m s^{-1} at z_R = 10 m, z_o = 0.5 m
Find: u_* = ? m s^{-1}

Use eq. (18.13):
u_* = (0.4)·(20 m s^{-1})/ln(10m/0.5m) = **2.67 m** s^{-1}

Check: Units OK. Magnitude OK.
Exposition: This corresponds to a large stress on the trees, which could make the branches violently move, causing some fruit to fall.

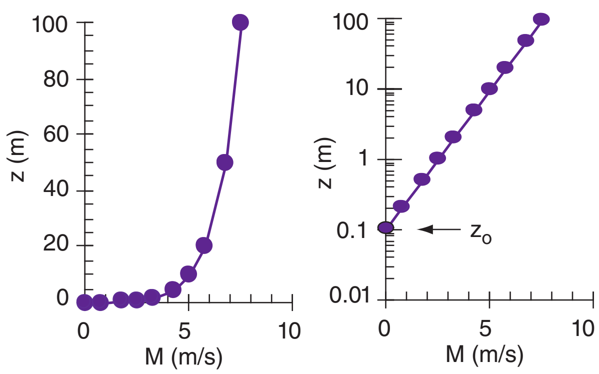
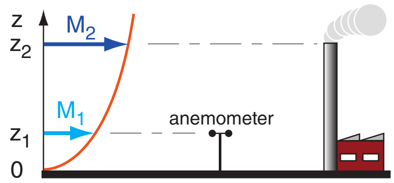


Figure 18.21
Wind-speed (M) profile in the statically neutral surface layer, for a roughness length of 0.1 m. (a) linear plot, (b) semi-log plot.

Sample Application
On an overcast day, a wind speed of 5 m s^{-1} is measured with an anemometer located 10 m above ground within an orchard. What is the wind speed at the top of a 25 m smoke stack?

Find the Answer
Given: M_1 = 5 m s^{-1} at z_1 = 10 m
Neutral stability (because overcast)
z_o = 0.5 m from Table 18-1 for an orchard
Find: M_2 = ? m s^{-1} at z_2= 25 m

Sketch:



Use eq. (18.14b):

$$M_2 = 5(\text{m/s})\cdot\frac{\ln(25\text{m}/0.5\text{m})}{\ln(10\text{m}/0.5\text{m})} = \mathbf{6.53\ m}\ \text{s}^{-1}$$

Check: Units OK. Physics OK. Sketch OK.
Exposition: Hopefully the anemometer is situated far enough from the smoke stack to measure the true undisturbed wind.

anemometer height for measuring "**surface winds**". For statically stable conditions, air becomes less turbulent and the drag coefficient decreases. For statically unstable conditions, use the relationships in section 10.3.5 to estimate turbulent drag.

Combining the previous two equations gives an expression for friction velocity in terms of surface wind speed and roughness length for statically neutral conditions:

$$u_* = \frac{k \cdot M_{10}}{\ln[z_R / z_o]} \qquad (18.13)$$

The physical interpretation is that faster winds over rougher surfaces causes greater kinematic stress.

18.5.3. Log Profile in the Neutral Surface Layer

Wind speed M is zero at the ground (more precisely, at a height equal to the aerodynamic roughness length). Speed increases roughly logarithmically with height in the statically neutral surface layer (bottom 50 to 100 m of the ABL), but the shape of this profile depends on the surface roughness:

$$M(z) = \frac{u_*}{k}\ln\left(\frac{z}{z_o}\right) \quad \text{for } z \ge z_o \qquad \bullet(18.14a)$$

Alternately, if you know wind speed M_1 at height z_1, then you can calculate wind speed M_2 at any other height z_2 :

$$M_2 = M_1 \cdot \frac{\ln(z_2 / z_o)}{\ln(z_1 / z_o)} \qquad (18.14b)$$

Many weather stations measure the wind speed at the standard height z_1 = 10 m.

An example of the **log wind profile** is plotted in Fig. 18.21. A perfectly logarithmic wind profile (i.e., eq. 18.14) would be expected only for **neutral** static stability (e.g., overcast and windy) over a uniform surface. For other static stabilities, the wind profile varies slightly from logarithmic.

On a semi-log graph, the log wind profile would appear as a straight line. You can determine the roughness length by measuring the wind speeds at two or more heights, and then extrapolating the straight line in a semi-log graph to zero wind speed. The z-axis intercept gives the roughness length.

18.5.4. Log-Linear Profile in Stable Surface Layer

During statically stable conditions, such as at nighttime over land, wind speed is slower near the ground, but faster aloft than that given by the log wind profile. This is because turbulence is weaker, causing less mixing and less homogenization of winds. The profile in the stable surface layer is empirically described by a **log-linear profile** formula with both a logarithmic and a linear term in z:

$$M(z) = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_o}\right) + 6\frac{z}{L}\right] \qquad \bullet(18.15)$$

where M is wind speed at height z, $k = 0.4$ is the von Kármán constant, z_o is the aerodynamic roughness length, and u_* is friction velocity. As height increases, the linear term dominates over the logarithmic term, as sketched in Fig. 18.20.

An **Obukhov length** L is defined as:

$$L = \frac{-u_*^{\,3}}{k\cdot(|g|/T_v)\cdot F_{Hsfc}} \qquad \bullet(18.16)$$

where $|g|$ = 9.8 m s^{-2} is gravitational acceleration magnitude, T_v is the absolute virtual temperature, and F_{Hsfc} is the kinematic surface heat flux. L has units of m, and is positive during statically stable conditions (because F_{Hsfc} is negative then). The Obukhov length can be interpreted as the height in the stable surface layer below which shear production of turbulence exceeds buoyant consumption.

18.5.5. Profile in the Convective Radix Layer

For statically unstable ABLs with vigorous convective thermals, such as occur on sunny days over land, wind speed becomes uniform with height a short distance above the ground. Between that uniform wind-speed layer and the ground is the **radix layer** (RxL). The wind speed profile in the radix layer is:

$$M(z) = M_{BL}\cdot\left(\zeta_*^{D}\right)^{A}\cdot\exp\left[A\cdot\left(1-\zeta_*^{D}\right)\right] \quad \text{for } 0 \le \zeta_* \le 1.0 \qquad \bullet(18.17a)$$

and

$$M(z) = M_{BL} \quad \text{for } 1.0 \le \zeta_* \qquad (18.17b)$$

where ζ_* = 1 defines the top of the radix layer. In the bottom of the RxL, wind speed increases faster with height than given by the log wind profile for the neutral surface layer, but becomes tangent to the uniform winds M_{BL} in the mid-mixed layer (Fig. 18.20).

The dimensionless height in the eqs. above is

$$\zeta_* = \frac{1}{C}\cdot\frac{z}{z_i}\cdot\left(\frac{w_*}{u_*}\right)^{B} \qquad (18.18)$$

where w_* is the **Deardorff velocity**, and the empirical coefficients are A = 1/4, B = 3/4, and C = 1/2. D = 1/2 over flat terrain, but increases to near D = 1.0 over hilly terrain.

The Deardorff velocity (eq. 3.39) is copied here:

$$w_* = \left[\frac{|g|}{T_v}\cdot z_i\cdot F_{Hsfc}\right]^{1/3} \qquad \bullet(18.19a)$$

Sample Application (§)

For a friction velocity of 0.3 m s^{-1}, aerodynamic roughness length of 0.02 m, average virtual temperature of 300 K, and kinematic surface heat flux of –0.05 K·m s^{-1} at night, plot the wind-speed profile in the surface layer. (Compare profiles for statically stable and neutral conditions.)

Find the Answer

Given: u_* = 0.3 m s^{-1}, z_o = 0.02 m,
T_v = 300 K, F_{Hsfc} = –0.05 K·m s^{-1}
Find: $M(z)$ = ? m s^{-1}

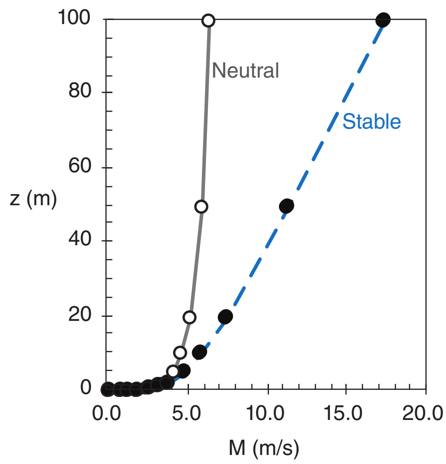
Use eq. (18.16):
L = –(0.3m s^{-1})^3/[0.4·(9.8m·s^{-2})·(–0.05K·m s^{-1})/(300K)]
= **<u>41.3 m</u>**

Use eq. (18.14a) for M in a neutral surface layer.
For example, at z = 50 m:
M = [(0.3m s^{-1})/0.4] · ln(50m/0.02m) = **<u>5.9 m</u>** s^{-1}

Use eq. (18.15) for M in a stable surface layer.
For example, at z = 50 m: M = [(0.3m s^{-1})/0.4] · [ln(50m/0.02m) + 6·(50m/41.3m)] = **<u>11.3 m</u>** s^{-1}

Use a spreadsheet to find M at the other heights:

z (m)	M(m s^{-1})$_{neutral}$	M (m s^{-1})$_{stable}$
0.02	0.0	0.0
0.05	0.7	0.7
0.1	1.2	1.2
0.2	1.7	1.7
0.5	2.4	2.5
1	2.9	3.0
2	3.5	3.7
5	4.1	4.7
10	4.7	5.7
20	5.2	7.4
50	5.9	11.3
100	6.4	17.3



Check: Units OK. Physics OK. Plot OK.
Exposition: Open circles are for neutral, solid are for statically stable. The linear trend is obvious in the wind profile for the stable boundary layer.

Sample Application (§)

For a 1 km deep mixed layer with surface heat flux of 0.3 K·m s^{-1} and friction velocity of 0.2 m s^{-1}, plot the wind speed profile using a spreadsheet. Terrain is flat, and mid-ABL wind is 5 m s^{-1}.

Find the Answer

Given: F_{Hsfc} =0.3 K·m s^{-1}, u_* = 0.2 m s^{-1},
z_i = 1000 m, M_{BL} = 5 m s^{-1}, D = 0.5 .
Find: $M(z)$ = ? m s^{-1} .

The ABL is statically <u>un</u>stable, because F_{Hsfc} is positive.
First, find w_* = ? m s^{-1} using eq. (18.19a).
Assume: $|g|/T_v$ = 0.0333 m·s^{-2}·K^{-1} (typical).

$$w_* = \left[\left(0.0333\frac{\text{m}}{\text{s}^2\text{K}}\right)\cdot(1000\text{m})\cdot\left(0.3\frac{\text{K}\cdot\text{m}}{\text{s}}\right)\right]^{1/3}$$

$$= (10\ \text{m}^3\ \text{s}^{-3})^{1/3} = 2.15\ \text{m s}^{-1}$$

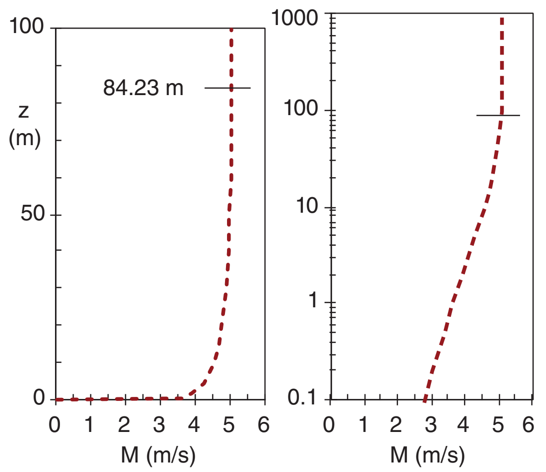
Use eq. (18.18) in a spreadsheet to get ζ_* at each z, then use eq. (18.17) to get each M. For example, at z = 10 m:

ζ_* = 2·(10m/1000m)·[(2.15m s^{-1})/(0.2m s^{-1})]$^{3/4}$
= <u>**0.1187**</u>, &

M =(5m s^{-1})·(0.119$^{1/2}$)$^{1/4}$·exp[0.25·(1– 0.119$^{1/2}$)]
= <u>**4.51m**</u> s^{-1}

z (m)	ζ_*	M (m s^{-1})
0	0.000	0.00
0.1	0.001	2.74
0.2	0.002	2.98
0.5	0.006	3.32
1.0	0.012	3.59
2	0.024	3.87
5	0.059	4.24
10	0.119	4.51
15	0.178	4.66
20	0.237	4.75

etc.



Check: Units OK. Physics OK. Sketch OK.
Exposition: This profile smoothly merges into the uniform wind speed in the mid-mixed layer, at height ζ_* = 1.0, which is at $z = C \cdot z_i \cdot (u_*/w_*)_B$ = 84.23 m from eq. (18.18).

where $|g|$ = 9.8 m s^{-2} is gravitational acceleration magnitude, T_v is absolute virtual temperature, z_i is depth of the ABL (= depth of the mixed layer), and F_{Hsfc} is the kinematic sensible heat flux (units of K·m s^{-1}) at the surface. Typical values of w_* are on the order of 1 m s^{-1}. The Deardorff velocity and buoyancy velocity w_B (defined in the Heat chapter) are both convective velocity scales for the statically unstable ABL, and are related by:

$$w_* \approx 0.08\ w_B \qquad (18.19b)$$

To use eq. (18.17) you need to know the average wind speed in the middle of the mixed layer M_{BL}, as was sketched in Fig. 18.9. The Atmos. Forces & Winds chapter shows how to estimate this if it is not known from measurements.

For both the free-convection radix layer and the forced-convection surface layer, turbulence transports momentum, which controls wind-profile shape, which then determines the shear (Fig. 18.22). However, differences in feedback lead to the difference between the radix layer and the surface layer.

In the neutral surface layer (Fig. 18.22a) there is strong feedback because wind shear generates the

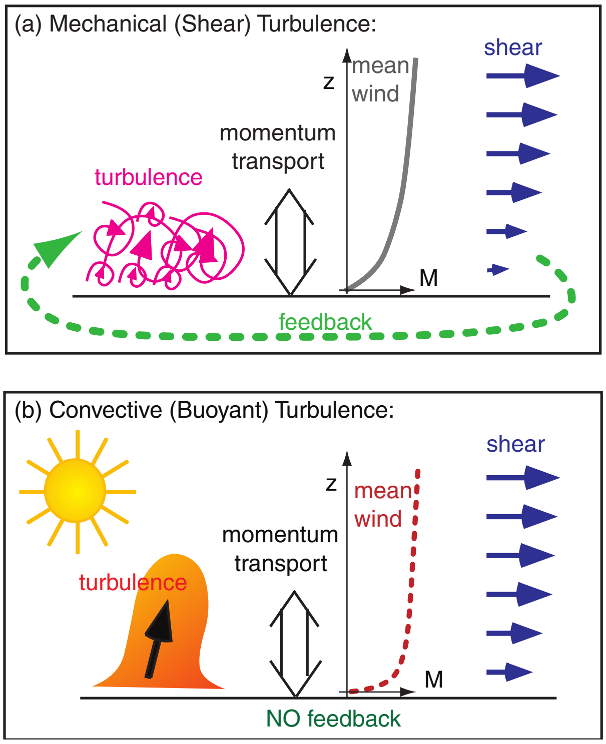


Figure 18.22
(a) Processes important for the log-wind profile in the surface layer dominated by mechanical turbulence (forced convection; neutral stability). (b) Processes important for the radix-layer wind profile during convective turbulence (free convection; statically unstable).

turbulence, which in turn controls the wind shear. However, such feedback is broken for convective turbulence (Fig. 18.22b), because it is generated primarily by buoyant thermals, not by shear.

18.6. TURBULENCE

18.6.1. Mean and Turbulent Parts

Wind can be quite variable. The total wind speed is the superposition of three types of flow:

- **mean wind** – relatively constant, but varying slowly over the course of hours;
- **waves** – regular (linear) oscillations of wind, often with periods of ten minutes or longer;
- **turbulence** – irregular, quasi-random, non-linear variations or gusts, with durations of seconds to minutes.

These flows can occur individually, or in any combination. Waves are discussed in the Regional Winds chapter. Here, we focus on mean wind and turbulence.

Let $U(t)$ be the x-direction component of wind at some instant in time, t. Different values of $U(t)$ can occur at different times, if the wind is variable. By averaging the **instantaneous wind** measurements over a time period, P, we can define a **mean wind** $\overline{U}$, where the overbar denotes an average. This mean wind can be subtracted from the instantaneous wind to give the **turbulence** or gust part u' (Fig. 18.23). Thus, the wind can be considered as a sum of mean and turbulent parts (neglecting waves for now).

Similar definitions exist for the other wind components (U, V, W), temperature (T) and humidity (r):

$$u'(t) = U(t) - \overline{U} \qquad (18.20a)$$

$$v'(t) = V(t) - \overline{V} \qquad (18.20b)$$

$$w'(t) = W(t) - \overline{W} \qquad (18.20c)$$

$$T'(t) = T(t) - \overline{T} \qquad (18.20d)$$

$$r'(t) = r(t) - \overline{r} \qquad (18.20e)$$

The averages in eq. (18.20) are defined over time or over horizontal distance. For example, the mean temperature is the sum of all individual temperature measurements, divided by the total number N of data points:

$$\overline{T} = \frac{1}{N}\sum_{k=1}^{N} T_k \qquad •(18.21)$$

Science Graffito

"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic."
– *Sir Horace Lamb* (1932)

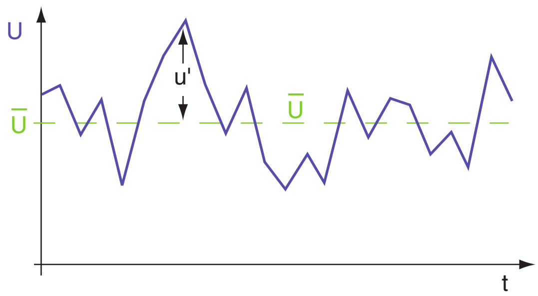


Figure 18.23
The instantaneous wind speed U is shown by the zigzag line. The average wind speed $\overline{U}$ is shown by the thin green horizontal dashed line. A gust velocity u' is the instantaneous deviation of the instantaneous wind from the average.

Sample Application

Given the following measurements of total instantaneous temperature, T, find the average $\overline{T}$. Also, find the T' values.

t (min)	T (°C)	t (min)	T (°C)
1	12	6	13
2	14	7	10
3	10	8	11
4	15	9	9
5	16	10	10

Find the Answer

As specified by eq. (18.21), adding the ten temperature values and dividing by ten gives the average $\overline{T}$ = 12.0°C. Subtracting this average from each instantaneous temperature gives:

t (min)	T '(°C)	t (min)	T '(°C)
1	0	6	1
2	2	7	–2
3	–2	8	–1
4	3	9	–3
5	4	10	–2

Check: The average of these T' values should be zero, by definition, useful for checking for mistakes.
Exposition: If a positive T' corresponds to a positive w', then warm air is moving up. This contributes positively to the heat flux.

where k is the data-point index (corresponding to different times or locations). The averaging time in eq. (18.21) is typically about 0.5 h. If you average over space, typical averaging distance is 50 to 100 km.

Short term fluctuations (described by the primed quantities) are associated with small-scale swirls of motion called **eddies**. The superposition of many such eddies of many sizes makes up the **turbulence** that is embedded in the mean flow.

Molecular viscosity in the air causes friction between the eddies, tending to reduce the turbulence intensity. Thus, turbulence is NOT a conserved quantity, but is **dissipative**. Turbulence decays and disappears unless there are active processes to generate it. Two such production processes are **convection**, associated with warm air rising and cool air sinking, and **wind shear**, the change of wind speed or direction with height.

Normally, weather forecasts are made for mean conditions, not turbulence. Nevertheless, the net effects of turbulence on mean flow must be included. Idealized average turbulence effects are given in the chapters on Thermodynamics, Water Vapor, and Atmos. Forces and Winds.

Meteorologists use statistics to quantify the net effect of turbulence. Some statistics are described next. In this chapter we will continue to use the overbar to denote the mean conditions. However, we drop the overbar in most other chapters in this book to simplify the notation.

Sample Application (§)

(a) Given the following V-wind measurements. Find the mean wind speed, and standard deviation.

(b) If the standard deviation of vertical velocity is 1 m s^{-1}, is the flow isotropic?

t (h)	V (m s^{-1})
0.1	2
0.2	–1
0.3	1
0.4	1
0.5	–3
0.6	–2
0.7	0
0.8	2
0.9	–1
1.0	1

Find the Answer

Given: Velocities listed at right

σ_w = 1 m s^{-1}.

Find: $\overline{V}$ = ? m s^{-1}, σ_v = ? m s^{-1}, isotropy = ?

(a) Use eq. (18.21), except for V instead of T:

$$\overline{V}(z) = \frac{1}{n}\sum_{i=1}^{n} V_i(z) = \frac{1}{10}(0) \quad = \underline{\mathbf{0\ m}}\ \mathrm{s}^{-1}$$

Use eq. (18.22), but for V:

$$\sigma_v{}^2 = \frac{1}{n}\sum_{i=1}^{n}(V_i - \overline{V})^2$$

$\sigma_v{}^2$ =(1/10)·(4+1+1+1+9+4+0+4+1+1) = 2.6 m^2 s^{-2}

Finally, use eq. (18.23), but for v:

$$\sigma_v = \sqrt{2.6 m^2 \cdot s^{-2}} \quad = \underline{\mathbf{1.61\ m}}\ \mathrm{s}^{-1}$$

Check: Units OK. Physics OK.

Exposition: (b) Anisotropic, because $\sigma_v > \sigma_w$ (see next subsection). This means that an initially spherical smoke puff would become elliptical in cross section as it disperses more in the horizontal than the vertical.

18.6.2. Variance and Standard Deviation

The **variance** σ^2 of wind speed is an overall statistic of gustiness. For example, for vertical velocity:

$$\begin{aligned}\sigma_w{}^2 &= \frac{1}{N}\sum_{k=1}^{N}(W_k - \overline{W})^2 \\ &= \frac{1}{N}\sum_{k=1}^{N}(w_k{}')^2 \\ &= \overline{w'^{\,2}}\end{aligned} \qquad \bullet(18.22)$$

Similar definitions can be made for $\sigma_u{}^2$, $\sigma_v{}^2$, $\sigma_\theta{}^2$, etc. Statistically, these are called "biased" variances. Velocity variances can exist in all three directions, even if there is a mean wind in only one direction.

The **standard deviation** σ is defined as the square-root of the variance, and can be interpreted as an average gust (for velocity), or an average turbulent perturbation (for temperatures and humidities, etc.). For example, standard deviations for vertical velocity, σ_w, and potential temperature, σ_θ , are:

$$\sigma_w = \sqrt{\sigma_w{}^2} = \overline{(w')^2}^{\,1/2} \qquad (18.23a)$$

$$\sigma_\theta = \sqrt{\sigma_\theta^2} = \overline{(\theta')^2}^{1/2} \tag{18.23b}$$

Larger variance or standard deviation of velocity means more intense turbulence.

For statically **stable** air, standard deviations in an ABL of depth h have been empirically found to vary with height z as:

$$\sigma_u = 2.2 \cdot u_* \cdot [1-(z/h)]^{3/4} \tag{18.24a}$$

$$\sigma_v = 2.2 \cdot u_* \cdot [1-(z/h)]^{3/4} \tag{18.24b}$$

$$\sigma_w = 1.73 \cdot u_* \cdot [1-(z/h)]^{3/4} \tag{18.24c}$$

where u_* is friction velocity. These equations work when the stability is weak enough that turbulence is not suppressed altogether.

For statically **neutral** air:

$$\sigma_u = 2.5 \cdot u_* \cdot \exp(-1.5 \cdot z/h) \tag{18.25a}$$

$$\sigma_v = 1.6 \cdot u_* \cdot [1-0.5 \cdot (z/h)] \tag{18.25b}$$

$$\sigma_w = 1.25 \cdot u_* \cdot [1-0.5 \cdot (z/h)] \tag{18.25c}$$

For statically **unstable** air:

$$\sigma_u = 0.032 \cdot w_B \cdot \left(1+[1-(z/z_i)]^6\right) \tag{18.26a}$$

$$\sigma_v = 0.032 \cdot w_B \tag{18.26b}$$

$$\sigma_w = 0.11 \cdot w_B \cdot (z/z_i)^{1/3} \cdot [1-0.8 \cdot (z/z_i)] \tag{18.26c}$$

where z_i is the mixed-layer depth, w_B is buoyancy velocity (eq. 3.38 or 18.19b). These relationships are important for air-pollution dispersion, and are used in the Air Pollution chapter. These equations are valid only within the boundary layer (i.e., from z = 0 up to h or z_i).

18.6.3. Isotropy

If turbulence has nearly the same variance in all three directions, then turbulence is said to be **isotropic**. Namely:

$$\sigma_u^2 \approx \sigma_v^2 \approx \sigma_w^2 \qquad \bullet(18.27)$$

[CAUTION: Do not confuse this word with "isentropic", which means adiabatic or of constant entropy.]

Sample Application (§)

Plot the vertical profiles of standard deviation of vertical velocity for statically stable and neutral situations for which the friction velocity is 0.5 m s^{-1}. The boundary-layer depth is 300 m.

Find the Answer

Given: u_* = 0.5 m s^{-1}, h = 300 m

Find: σ_w = ? vs. z for neutral and stable conditions

Use eqs. (18.24c and 18.25c). For example, for z = 100m:

Stable:

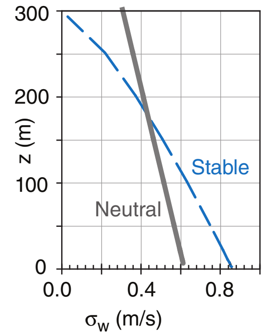
σ_w = 1.73·(0.5m s^{-1})·[1–(100m/300m)]$^{0.75}$ = **0.638 m** s^{-1}

Neutral:

σ_w = 1.25·(0.5m s^{-1})·[1–0.5·(100m/300m) = **0.521 m** s^{-1}

Using a spreadsheet with a range of heights yields:

	sigma w (m/s)	
z (m)	Stable	Neut.
0.0	0.865	0.625
1.0	0.863	0.624
10	0.843	0.615
100	0.638	0.521
150	0.514	0.469
200	0.379	0.417
250	0.266	0.365
300	0.0	0.313



Check: Units OK. Magnitudes OK.

Exposition: Turbulence diminishes rapidly with increasing height for statically stable conditions.

Sample Application (§)

For a 1 km thick unstable ABL, compare the vertical profiles of σ_u & σ_w. Assume w_B = 30 m s^{-1}.

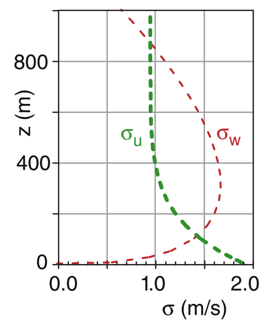
Find the Answer

Given: z_i = 1000 m , w_B = 30 m s^{-1}

Find: σ_u & σ_w vs. z .

Use eqs. (18.26a and 18.26c) in a spreadsheet, and plot.

	sigma (m/s)	
z (m)	σ_u	σ_w.
0	1.920	0.0
50	1.666	1.167
100	1.470	1.409
200	1.212	1.621
300	0.514	1.679
500	0.975	1.572
800	0.960	1.103
1000	0.960	0.660



Check: Units OK. Magnitude reasonable.

Exposition: The flow is anisotropic at most heights. Near both the surface and z_i, horizontal turbulence is greater than vertical. But in the middle of the mixed layer, vertical turbulence is greater than horizontal.

Sample Application

Find u_*, the velocity standard deviations, and *TKE* in statically stable air at height 50 m in an ABL that is 200 m thick. Assume C_D = 0.002, and the winds at height 10 m are 5 m s^{-1}.

Find the Answer

Given: z = 50 m, h = 200 m, C_D = 0.002,
M = 5 m s^{-1}. Statically stable.
Find: u_*, σ_u, σ_v, σ_w = ? m s^{-1}. *TKE* = ? m^2 s^{-2}.

Use eq. (18.11):
$u_*^2 = 0.002\cdot(5\text{m s}^{-1})^2 = 0.05$ m^2 s^{-2}. u_* = **0.22 m** s^{-1}

Use eqs. (18.24a-c):
$\sigma_u = 2\cdot(0.22\text{m s}^{-1})\cdot[1-(50\text{m}/200\text{m})]^{3/4}$ = **0.35 m** s^{-1}

$\sigma_v = 2.2\cdot(0.22\text{m s}^{-1})\cdot[1-(50\text{m}/200\text{m})]^{3/4}$ = **0.39 m** s^{-1}

$\sigma_w = 1.73\cdot(0.22\text{m s}^{-1})\cdot[1-(50\text{m}/200\text{m})]^{3/4}$ = **0.31 m** s^{-1}

Use eq. (18.28b):

$TKE = 0.5\cdot[0.35^2 + 0.39^2 + 0.31^2]$ = **0.185** m^2 s^{-2}.

Check: Units OK. Physics OK.
Exposition: In statically stable air, vertical turbulence is generally less than horizontal turbulence. Also, turbulence intensity increases with wind speed. If the atmosphere is too stable, then there will be no turbulence (see "dynamic stability" in the Atmospheric Stability chapter).

Science Graffito

"Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity – in the molecular sense."
– *L.F. Richardson*, 1922: "Weather Prediction by Numerical Process". Cambridge Univ. Press. p66.

Turbulence is **anisotropic** (not isotropic) in many situations. During the daytime over bare land, rising thermals create stronger vertical motions than horizontal. Hence, a smoke puff becomes **dispersed** (i.e., spread out) in the vertical faster than in the horizontal. At night, vertical motions are very weak, while horizontal motions can be larger. This causes smoke puffs to **fan** out horizontally with only little vertical dispersion in statically stable air.

18.6.4. Turbulence Kinetic Energy

An overall measure of the intensity of turbulence is the **turbulence kinetic energy** per unit mass (*TKE*):

$$TKE = 0.5\cdot\left[\overline{(u')^2} + \overline{(v')^2} + \overline{(w')^2}\right] \qquad \bullet(18.28a)$$

$$TKE = 0.5\cdot\left[\sigma_u^{\,2} + \sigma_v^{\,2} + \sigma_w^{\,2}\right] \qquad \bullet(18.28b)$$

TKE is usually produced at eddy sizes that scale to the boundary-layer depth. The production is done mechanically by wind shear and buoyantly by thermals.

Turbulent energy cascades through the **inertial subrange**, where the large-size eddies drive medium ones, which in turn drive smaller eddies. Molecular viscosity continuously damps the tiniest (**microscale**) eddies, dissipating *TKE* into heat. *TKE* is not conserved.

The tendency of *TKE* to increase or decrease is given by the following *TKE* budget equation:

$$\frac{\Delta TKE}{\Delta t} = A + S + B + Tr - \varepsilon \qquad \bullet(18.29)$$

where A is advection of *TKE* by the mean wind, S is shear generation, B is buoyant production or consumption, Tr is transport by turbulent motions and pressure, and ε is viscous dissipation rate. For **stationary** (steady-state) turbulence, the tendency term on the left side of eq. (18.29) is zero.

Mean wind blows *TKE* from one location to another. The **advection** term is given by:

$$A = -U\cdot\frac{\Delta TKE}{\Delta x} - V\cdot\frac{\Delta TKE}{\Delta y} - W\cdot\frac{\Delta TKE}{\Delta z} \qquad (18.30)$$

Thus, turbulence can increase (or decrease) at any location if the wind is blowing in greater (or lesser) values of TKE from somewhere else.

Wind shear generates turbulence near the ground according to:

$$S = u_*^2 \cdot \frac{\Delta M}{\Delta z} \tag{18.31a}$$

in the surface layer, where u_* is the friction velocity, and $\Delta M/\Delta z$ is the wind shear. To good approximation for near-neutral static stability:

$$S \approx a \cdot M^3 \tag{18.31b}$$

where $a = 2\text{x}10^{-4}\ \text{m}^{-1}$ for wind speed M measured at a standard height of z = 10 m. Greater wind speeds near the ground cause greater wind shear, and generate more turbulence.

Buoyancy can either increase or decrease turbulence. When thermals are rising from a warm surface, they generate *TKE*. Conversely, when the ground is cold and the ABL is statically stable, buoyancy opposes vertical motion and consumes *TKE*.

The rate of **buoyant production or consumption** of *TKE* is:

$$B = \frac{|g|}{T_v} \cdot F_{H\ sfc} \tag{18.32}$$

where $|g|$= 9.8 m·s^{-2} is gravitational acceleration magnitude, T_v is the absolute virtual air temperature near the ground, and $F_{H\ sfc}$ is the kinematic effective surface heat flux (positive when the ground is warmer than the air). Over land, $F_{H\ sfc}$ and B are usually positive during the daytime, and negative at night.

Turbulence can advect or **transport** itself. For example, if turbulence is produced by shear near the ground (in the **surface layer**), then turbulence motions will tend to move the excess *TKE* from the surface layer to locations higher in the ABL. Pressure fluctuations can have a similar effect, because turbulent pressure forces can generate turbulence motions. This pressure term is difficult to simplify, and will be grouped with the turbulent transport term, *Tr*, here.

Molecular viscosity dissipates turbulent motions into heat. The amount of heating is small, but the amount of damping of *TKE* is large. The **dissipation** is always a loss:

$$\varepsilon \approx \frac{(TKE)^{3/2}}{L_\varepsilon} \tag{18.33}$$

where $L_\varepsilon \approx 50$ m is a **dissipation length scale**.

The ratio of buoyancy to shear terms of the *TKE* equation is called the **flux Richardson number**, R_f:

$$R_f = \frac{-B}{S} \approx \frac{-(|g|/T_v)\cdot F_{H\ sfc}}{u_*^2 \cdot \dfrac{\Delta M}{\Delta z}} \tag{18.34a}$$

HIGHER MATH • Shear Generation

To get *TKE* shear-generation eq. (18.31b), start with eq. (18.31a):

$$S = u_*^2 \cdot (\partial M/\partial z) \tag{18.31a}$$

But

$$(\partial M/\partial z) = u_*/(k\cdot z) \qquad \text{from eq. (18.14a).}$$

Thus,

$$S = u_*^3 \ / \ (k\cdot z)$$

But

$$u_*^2 = C_D \cdot M^2 \qquad \text{from eq. (18.11)}$$

which gives

$$u_*^3 = C_D^{3/2} \cdot M^3$$

Thus:

$$S = [C_D^{3/2}/(k\cdot z)] \cdot M^3$$

or

$$S = a \cdot M^3 \tag{18.31b}$$

where

$$a = [C_D^{3/2}/(k\cdot z)]$$

For $C_D \approx 0.01$, k = 0.4 (von Kármáns constant), and z = 10 m, the result is $a = 2.5\text{x}10^{-4}\ \text{m}^{-1}$. For any wind speed, shear generation can vary by an order of magnitude depending on the drag coefficient and height.

Sample Application

Assume steady state, and neglect advection and transport. What equilibrium TKE is expected in the surface layer with a mean wind of 5 m s^{-1} and surface heat flux of –0.02 K·m s^{-1}? The ambient temperature is 25°C, and the air is dry.

Find the Answer

Given: M = 5 m s^{-1}, $F_{H\ sfc}$ = –0.02 K·m s^{-1}, A = 0, Tr = 0, $\Delta TKE/\Delta t = 0$ for steady state, T_v = 298 K.
Find: TKE = ? m^2 s^{-2} .

Rearrange eq. (18.29).

$$\varepsilon = S + B$$

But ε depends on *TKE*, thus, we can rearrange eq. (18.33) to solve for $TKE = (L_\varepsilon \cdot [\varepsilon])^{2/3}$ and then plug in the eq. above:

$$TKE = \{ L_\varepsilon \cdot [S + B] \}^{2/3}$$

Use eqs. (18.31b and 18.32) to find S and B, and plug into the equation above:

$$TKE = \left\{ L_\varepsilon \cdot \left[a\cdot M^3 + (|g|/T_v)\cdot F_{H\ sfc} \right] \right\}^{2/3} = \Bigg\{ (50\text{m}) \cdot$$

$$\left[\left(2 \cdot 10^{-4}\text{m}^{-1}\right)\left(5\frac{\text{m}}{\text{s}}\right)^3 + \frac{9.8\text{ms}^{-2}}{298\text{K}}(-0.02\text{Km/s}) \right] \Bigg\}^{2/3}$$

$= \{1.25 – 0.033\ \text{m}^3\ \text{s}^{-3}\}^{2/3}$ = **<u>1.14</u>** m^2 s^{-2}

Check: Units OK. Physics OK.
Exposition: This turbulence intensity is weak, as is typical at night when heat fluxes are negative. Also, eq. (18.31b) for S is not accurate for statically stable conditions.

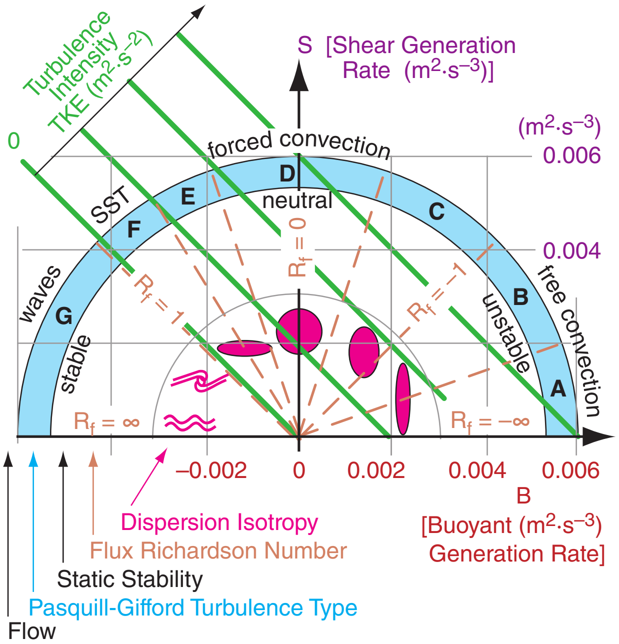


Figure 18.24
Rate of generation of TKE by buoyancy (abscissa) and shear (ordinate). Shapes and relative rates of plume dispersion are suggested by the dark magenta spots and waves. Dashed lines separate sectors of different Pasquill-Gifford turbulence type (A - G). Isopleths of TKE intensity are given by the green diagonal lines. R_f is flux Richardson number. SST is stably-stratified turbulence.

Sample Application
For the previous Sample Application, determine the nature of convection (free, forced, etc.), the Pasquill-Gifford (PG) turbulence type, and the flux Richardson number. Assume no clouds.

Find the Answer
Given: (see previous Sample Application)
Find: $S = ?\ m^2 s^{-3}$, $B = ?\ m^2 s^{-3}$, $R_f = ?$, PG = ?

Use eq. (18.31b):

$$S \approx \left(2 \cdot 10^{-4} m^{-1}\right)\left(5\frac{m}{s}\right)^3 = 0.025\ m^2\ s^{-3}$$

Use eq. (18.32):

$$B = \frac{9.8 ms^{-2}}{298K}(-0.02 Km/s) = -0.00066\ \ m^2\ s^{-3}$$

Because the magnitude of B is less than a third of that of S, we conclude convection is **forced**.

Use eq. (18.34): $R_f = -(-0.00066) / 0.025 =$ **0.0264** and is dimensionless.

Use Fig. 18.24. Pasquill-Gifford Type = **D** (but on the borderline near E).

Check: Units OK. Physics OK.
Exposition: The type of turbulence is independent of the intensity. Intensity is proportional to S+B.

$$R_f \approx \frac{-(|g|/T_v)\cdot F_{H\ sfc}}{a\cdot M^3} \qquad (18.34b)$$

with $a \approx 2\text{x}10^{-4}\ m^{-1}$. R_f is approximately equal to the **gradient or bulk Richardson number**, discussed in the Stability chapter. Generally, turbulence dies if $R_f > 1$.

18.6.5. Free and Forced Convection

The nature of turbulence, and therefore the nature of pollutant dispersion, changes with the relative magnitudes of terms in the *TKE* budget. Two terms of interest are the shear (S) and buoyancy (B) terms.

When $|B| < |S/3|$, the atmosphere is said to be in a state of **forced convection** (Fig. 18.24). These conditions are typical of windy overcast days, and are associated with near **neutral static stability**. Turbulence is nearly **isotropic**. Smoke plumes disperse at nearly equal rates in the vertical and lateral, which is called **coning**. The sign of B is not important here — only the magnitude.

When B is positive and $|B| > |3{\cdot}S|$, the atmosphere is said to be in a state of **free convection**. **Thermals** of warm rising air are typical in this situation, and the ABL is **statically unstable** (in the nonlocal sense; see the Stability chapter). These conditions often happen in the daytime over land, and during periods of cold-air advection over warmer surfaces. Turbulence is **anisotropic**, with more energy in the vertical, and smoke plumes loop up and down in a pattern called **looping**.

When B is negative and $|B| > |S|$, static stability is so strong that turbulence cannot exist. During these conditions, there is virtually no dispersion while the smoke blows downwind. **Buoyancy waves** (**gravity waves**) are possible, and appear as waves in the smoke plumes. For values of $|B| \approx |S|$, breaking **Kelvin-Helmholtz waves** can occur (see the Stability chapter), which cause some dispersion.

When B is negative but $|B| < |S|$, weak turbulence is possible. These conditions can occur at night. This is sometimes called **stably-stratified turbulence** (**SST**). Vertical dispersion is much weaker than lateral, causing an **anisotropic** condition where smoke spreads horizontally more than vertically, in a process called **fanning**.

Fig. 18.24 shows the relationship between different types of convection and the terms of the *TKE* equation. While the ratio of B/S determines the nature of convection, the sum $S + B$ determines the intensity of turbulence. A Pasquill-Gifford turbulence type (Fig. 18.24) can also be defined from the relative magnitudes of S and B, and is used in the Air Pollution chapter to help estimate pollution dispersion rates.

18.6.6. Turbulent Fluxes and Covariances

Rewrite eq. (18.22) for variance of w as

$$\mathrm{var}(w) = \frac{1}{N}\sum_{k=1}^{N}(W_k - \overline{W})\cdot(W_k - \overline{W}) \qquad (18.35)$$

By analogy, a **covariance** between vertical velocity w and potential temperature θ can be defined as:

$$\mathrm{covar}(w,\theta) = \frac{1}{N}\sum_{k=1}^{N}(W_k - \overline{W})\cdot(\theta_k - \overline{\theta})$$

$$= \frac{1}{N}\sum_{k=1}^{N}(w_k{'})\cdot(\theta_k{'}) \qquad \bullet(18.36)$$

$$= \overline{w'\theta'}$$

where the overbar still denotes an average. Namely, one over N times the sum of N terms (see middle line of eq. 18.36) is the average of those items. Comparing eqs. (18.35) with (18.36), we see that variance is just the covariance between a variable and itself.

Covariance indicates the amount of common variation between two variables. It is positive where both variables increase and/or decrease together. Covariance is negative for opposite variation, such as when one variable increases while the other decreases. Covariance is zero if one variable is unrelated to the variation of the other.

The **correlation coefficient** $r_{a,b}$ is defined as the covariance between a and b, normalized by the standard deviations of those two variables. By normalized, we mean that $-1 \le r_{a,b} \le 1$. Using vertical velocity and potential temperature for illustration:

$$r_{w,\theta} = \frac{\overline{w'\theta'}}{\sigma_w \cdot \sigma_\theta} \qquad \bullet(18.37)$$

A correlation coefficient of +1 indicates a perfect correlation (both variables increase or decrease together proportionally), –1 indicates perfect opposite correlation, and zero indicates no correlation. Because it is normalized, $r_{a,b}$ gives no information on the absolute magnitudes of the variations.

In the ABL, many turbulent variables are correlated. For example, in the statically unstable ABL (Fig. 18.25a), parcels of warm air rise and while other cool parcels sink in convective circulations. Warm air (θ' = +) going up (w' = +) gives a positive product [$(w'\theta')_{up}$ = +]. Cool air (θ' = –) going down (w' = –) also gives a positive product [$(w'\theta')_{down}$ = +].

The average of those two products is also positive [$\overline{w'\theta'} = 0.5\cdot((w'\theta')_{up} + (w'\theta')_{down}) = +$]. The result gives positive correlation coefficients $r_{w,\theta}$ during free convection, which is typical during daytime.

Science Graffito

Seen on a bumper sticker:
"**Lottery**: A tax on people who are bad at math."

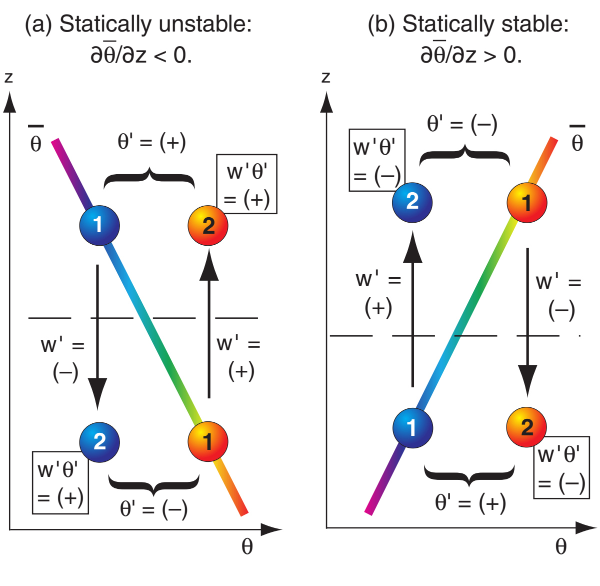


Figure 18.25
(a) Relationship between turbulent potential temperature θ and vertical velocity w for a statically unstable environment (e.g., daytime over land with clear skies) . (b) Same, but for a statically stable environment (e.g., nighttime over land with clear skies). In both figures, the thick line represents the ambient environment, circles represent air parcels, with orange being the warm air parcel and blue being cool. Numbers 1 and 2 indicate starting and ending positions of each air parcel during time interval Δt. (+) or (–) mean greater or less than zero, respectively.

Sample Application (§)

Fast-response measurements of potential temperature θ, water-vapor mixing ratio *r*, and *u* and *w* components of wind are given below as a function of time *t*. For θ and *w*, find their means, variances, and standard deviations. Also find the covariance, correlation coefficient, kinematic heat flux, and the heat flux (W m^{-2}).

Columns C and D are not used in this example, but will be used in some of the homeworks.

	A	B	C	D	E
1	Given:				
2	t (s)	θ (°C)	r (g/kg)	U (m/s)	W (m/s)
3	0	21	6.0	10	-5
4	0.1	28	9.5	6	4
5	0.2	29	10.0	7	3
6	0.3	25	8.0	3	4
7	0.4	22	6.5	5	0
8	0.5	28	9.5	15	-5
9	0.6	23	7.0	12	-1
10	0.7	26	8.5	16	-3
11	0.8	27	9.0	10	2
12	0.9	24	7.5	8	-4
13	1.0	21	6.0	14	-4
14	1.1	24	7.5	10	1
15	1.2	25	8.0	13	-2
16	1.3	27	9.0	5	3
17	1.4	29	10.0	7	5
18	1.5	22	6.5	11	2
19	1.6	30	10.5	2	6
20	1.7	23	7.0	15	-1
21	1.8	28	9.5	13	3
22	1.9	21	6.0	12	-3
23	2.0	22	6.5	16	-5
24	**avg =**	**25**			**0**

Find the Answer:

Given: Data above in rows 2 through 23.

Find: $\overline{W}$ =? m s^{-1} , $\overline{\theta}$ = ? °C , σ_w^2 = ? m^2 s^{-2},
σ_θ^2 = ? °C^2, σ_w =? m s^{-1}, σ_θ = ? °C,
$\overline{w'\theta'}$ = ? K·m s^{-1}, $r_{w,\theta}$ = ? (dimensionless)
F_H = ? K·m s^{-1}, $\mathbb{F}_H$ = ? W m^{-2} .

First, use eq. (18.21) to find the mean values. These answers are already shown in row 24 above.

$\overline{\theta}$ = **25 °C** , $\overline{W}$ = **0 m** s^{-1}

Next, use eqs. (18.20) to find the deviation from the mean, for each of the observations. The results are tabulated in columns G and H on the next page. Then square each of those perturbation (primed) values, as tabulated in columns I and J on the next page.

Use eq. (18.22), averaging the squared perturbations to give the variances (row 24, columns I and J):

σ_θ^2 = **8.48 °C^2** , σ_w^2 = **12.38** m^2 s^{-2}.

The square root of those answers (eq. 18.23) gives the standard deviations in row 25, columns I and J:

σ_θ = **2.91 °C** , σ_w = **3.52 m** s^{-1} .

(continues on next page)

Conversely, for statically stable conditions (Fig. 18.25b) where wind-shear-induced turbulence drives vertical motions against the restoring buoyant forces (see the Stability chapter), one finds cold air moving up, and warm air moving down. (Parcel warmth or coldness is measured relative to the ambient mean potential temperature $\overline{\theta}$ at the same ending height as the parcel.) This gives $\overline{w'\theta'} < 0$ (i.e., $\overline{w'\theta'} = -$), which is often the case during night.

More important than the statistics are the physical processes they represent. Covariances represent fluxes. Look at the air parcels crossing the horizontal dashed line in Fig. 18.25a. Pretend that the dashed line is an edge view of a horizontal area of 1 m^2.

During time interval Δ*t*, the warm (light-grey shaded) air parcel moves warm air upward through that area in Fig. 18.25a. Heat flux is defined as heat moved per area per time. Thus this rising warm air parcel contributes to a positive heat flux. Similarly, the cold sinking air parcel (shaded dark grey) contributes to a positive heat flux through that area (because negative w' times negative θ' is positive). Both parcels contribute to a positive heat flux.

This implies that covariance between vertical velocity and potential temperature is a turbulent kinematic heat flux, F_H [= $F_{z\ turb}(\theta)$ in the notation of the Heat chapter]:

$$\overline{w'\theta'} = F_H \qquad \bullet(18.38a)$$

Similarly, the covariance between vertical velocity *w* and water vapor mixing ratio, *r* (see the Water Vapor chapter), is a kinematic moisture flux, $F_{z\ turb}(r)$:

$$\overline{w'r'} = F_{z\ turb}(r) \qquad \bullet(18.38b)$$

Momentum flux is even more interesting. Recall from physics that **momentum** is mass times velocity. Units would be kg·m·s^{-1}. Therefore, momentum flux (momentum per area per time) would have units of (kg·m·s^{-1})·m^{-2}·s^{-1} = (kg·m·s^{-2})·m^{-2} = N·m^{-2}. [see Appendix A to relate a force of 1 Newton (N) to other units]. But N·m^{-2} is a force per unit area, which is the definition of stress, τ. Thus, stress and momentum flux are physically the same in a fluid such as air.

A kinematic momentum flux is the momentum flux divided by air density ρ, which from the paragraph above is equal to τ/ρ. But this is just the definition of friction velocity squared u_*^2 (eq. 18.10).

As in Fig. 18.25, if vertically moving air parcels (w') transport air with different horizontal velocities (u') across a horizontal area per unit time, then the covariance between vertical velocity and horizontal velocity is a kinematic momentum flux. From the paragraph above, the magnitude is also equal to the friction velocity squared. Thus:

•(18.38c)

$$\left|\overline{w'u'}\right| = \left|F_{z\ turb}(\text{momentum})\right| = \left|\tau / \rho\right| = u_*^{\,2}$$

where $\overline{w'u'}$ is called a **Reynolds stress**.

In Fig. 18.25, air mass is conserved. Namely, each rising air parcel is compensated by a descending air parcel with the same air mass. Thus, as seen from the discussion above, turbulence can cause a net vertical transport of heat, moisture, and momentum, even though there is no net transport of air mass.

Turbulent fluxes given by eqs. (18.38) are called **eddy-correlation fluxes**. They can be measured with fast response velocity, humidity, and temperature sensors, sampling at about 10 Hz for 30 minutes. Turbulent fluxes can also be parameterized, as discussed next.

18.6.7. Turbulence Closure

To forecast the weather (see the NWP chapter), we need to solve the Eulerian conservation equations for temperature, humidity, and wind:

- temperature forecasts ← heat conservation eq. ← First Law of Thermodynamics (see the Thermodynamics chapter)
- humidity forecasts ← water conservation eq. ← Eulerian water-budget equation (see the Water Vapor chapter)
- wind forecasts ← momentum conservation eq. ← Newton's Second Law (see the Atmos. Forces & Winds chapter)

For example, the Eulerian net heat-budget equation from the Thermodynamics chapter (eq. 3.51) is:

(18.39a)

$$\frac{\Delta T}{\Delta t} = Advection + Radiation + LatentHeat - \frac{\Delta F_{z\ turb}(\theta)}{\Delta z}$$

where the last term is the turbulence term. But from eq. (18.38a), we recognize the turbulent heat flux as a covariance. Thus, eq. (18.39a) can be rewritten as:

$$\frac{\Delta \overline{T}}{\Delta t} = (otherPhysics) - \frac{\Delta \overline{w'\theta'}}{\Delta z} \qquad (18.39b)$$

The derivation of this equation is shown in the HIGHER MATH box on the next page.

Because the heat flux $\overline{w'\theta'}$ is needed in eq. (18.39b), we need to get a forecast equation for it:

$$\frac{\Delta \overline{w'\theta'}}{\Delta t} = (otherPhysics) - \frac{\Delta \overline{w'w'\theta'}}{\Delta z} \qquad (18.40)$$

Sample Application (§) *(continuation)*

Use eq. (18.36) and multiply each w' with θ' to give in column K the values of $w'\theta'$. Average those to get the covariance: $\overline{w'\theta'} = F_H =$ **6.62 K·m** s^{-1}, which is the kinematic heat flux by definition.

Use eq. (18.37) and divide the covariance by the standard deviations to give the correlation coef: $r_{w,\theta}$ = **0.65** (dimensionless).

Use eq. (2.11) to give heat flux, with $\rho \cdot C_p$ =1231 (W m^{-2})/(°C·m s^{-1}) from Appendix B, yielding $\mathbb{F}_H = \rho \cdot C_p \cdot F_H$ = **8150 W m^{-2}** .

	G	H	I	J	K	L
1	θ'	w'	θ'^{2}	w'^{2}	w'θ'	
2	(°C)	(m/s)	(°C^2)	(m/s)^2	°C·(m/s)	
3	–4	–5	16	25	20	
4	3	4	9	16	12	
5	4	3	16	9	12	
6	0	4	0	16	0	
7	–3	0	9	0	0	
8	3	–5	9	25	–15	
9	–2	–1	4	1	2	
10	1	–3	1	9	–3	
11	2	2	4	4	4	
12	–1	–4	1	16	4	
13	–4	–4	16	16	16	
14	–1	1	1	1	–1	
15	0	–2	0	4	0	
16	2	3	4	9	6	
17	4	5	16	25	20	
18	–3	2	9	4	–6	
19	5	6	25	36	30	
20	–2	–1	4	1	2	
21	3	3	9	9	9	
22	–4	–3	16	9	12	
23	–3	–5	9	25	15	
24	**0**	**var.=**	8.48	12.38	**6.62=**	**covar**
25		**st.dev.=**	2.61	3.52	**0.65=**	**$r_{w,\theta}$**

Check: The average is zero of the singled primed values, as they always should be. Units OK.

Exposition: The magnitude of $\mathbb{F}_H$ is unrealistically big for this contrived data set.

Sample Application

If $\overline{w'\theta'}$ = 0.2 K·m s^{-1} at the surface, and is 0 at the top of a 1 km thick layer, find the warming rate.

Find the Answer

Given: $\overline{w'\theta'}$ = 0.2 K·m s^{-1} at z = 0, $\overline{w'\theta'}$ =0 at z = 1km

Find: $\Delta \overline{T} / \Delta t$ = ? K h^{-1}

Use eq. (18.39b):

$\Delta \overline{T} / \Delta t$ = – (0 – 0.2 K·m s^{-1}) /(1000m – 0)

= 2x10^{-4} K s^{-1} = **0.72 K** h^{-1}.

Check: Units OK. Magnitude small.

Exposition: Over 12 hours of daylight, this surface flux would warm the thick layer of air by 8.6°C.

HIGHER MATH • Turbulence Terms

Why does a turbulence covariance term appear in the forecast equation for average temperature, $\overline{T}$? To answer, consider the vertical advection term (3.31) in the heat-budget equation (3.17) as an example:

$$\frac{\partial T}{\partial t} = \cdot\cdot\cdot - W\frac{\partial \theta}{\partial z}$$

For each dependent variable (*T*, *W*, θ), describe them by their mean plus turbulent parts:

$$\frac{\partial(\overline{T}+T')}{\partial t} = \cdot\cdot\cdot - (\overline{W}+w')\frac{\partial(\overline{\theta}+\theta')}{\partial z}$$

or

$$\frac{\partial \overline{T}}{\partial t} + \frac{\partial T'}{\partial t} = \cdot\cdot\cdot - \overline{W}\frac{\partial \overline{\theta}}{\partial z} - \overline{W}\frac{\partial \theta'}{\partial z} - w'\frac{\partial \overline{\theta}}{\partial z} - w'\frac{\partial \theta'}{\partial z}$$

Next, average the whole equation. But the average of a sum is the same as the sum of the averages:

$$\frac{\partial \overline{\overline{T}}}{\partial t} + \frac{\partial \overline{T'}}{\partial t} = \cdot\cdot\cdot - \overline{\overline{W}\frac{\partial \overline{\theta}}{\partial z}} - \overline{\overline{W}\frac{\partial \theta'}{\partial z}} - \overline{w'\frac{\partial \overline{\theta}}{\partial z}} - \overline{w'\frac{\partial \theta'}{\partial z}}$$

[**Aside**: Let *A* be any variable. Expand into mean and turbulent parts: $A = \overline{A} + a'$, then average the whole eq: $\overline{A} = \overline{\overline{A}} + \overline{a'}$. But the average of an average is just the original average: $\overline{A} = \overline{A} + \overline{a'}$. This equation can be valid only if $\overline{a'} = 0$. Thus, the average of any term containing a single primed variable (along with any number of unprimed variables) is zero.]

Thus the heat budget becomes:

$$\frac{\partial \overline{\overline{T}}}{\partial t} = \cdot\cdot\cdot - \overline{\overline{W}\frac{\partial \overline{\theta}}{\partial z}} - \overline{w'\frac{\partial \theta'}{\partial z}}$$

The term on the left and the first term on the right are averages of averages, and can be rewritten as just the original averages. The last term can be transformed into **flux form** using the turbulent continuity equation (which works if you apply it to the turbulent advection in all 3 directions, but which is not shown here). The end result is:

$$\frac{\partial \overline{T}}{\partial t} = \cdot\cdot\cdot - \overline{W}\frac{\partial \overline{\theta}}{\partial z} - \frac{\partial \overline{w'\theta'}}{\partial z}$$

This says that to forecast the average temperature, you need to consider not only the average advection by the mean wind (first term on the right), but you also need to consider the turbulence flux divergence (last term on the right).

Similar terms appear for advection in the *x* and *y* directions. Also, similar terms appear in the forecast equations for moisture and wind. Thus, the effects of turbulence cannot be neglected.

To simplify the notation in almost all of this book, the overbar is left off of the terms for mean temperature, mean wind, etc. Also, earlier in this chapter, and in other chapters, the turbulence flux divergence term was parameterized directly as a function of non-turbulent (average) wind, temperature, humidity, etc. Such parameterizations are turbulence closure approximations.

But this contains yet another unknown $\overline{w'w'\theta'}$. A forecast equation for $\overline{w'w'\theta'}$ would yield yet another unknown. Hence, we need an infinite number of equations just to forecast air temperature. Or, if we use only a finite number of equations, then we have more unknowns than equations.

Hence, this set of equations is mathematically not closed, which means they cannot be solved. To be a **closed system of equations**, the number of unknowns must equal the number of equations.

One reason for this **closure problem** is that it is impossible to accurately forecast each swirl and eddy in the wind. To work around this problem, meteorologists **parameterize** the net effect of all the eddies; namely, they use a finite number of equations, and approximate the unknowns as a function of known variables. Such an approximation is called **turbulence closure**, because it mathematically closes the governing equations, allowing useful weather forecasts and engineering designs.

18.6.7.1. Turbulence Closure Types

For common weather situations with mean temperature, wind and humidity that are nearly horizontally uniform, turbulent transport in any horizontal direction nearly cancels transport in the opposite direction, and thus can be neglected. But vertical transport is significant. Medium- and large-sized turbulent eddies can transport air parcels from many different source heights to any destination height within the turbulent domain, where the smaller eddies mix the parcels together.

Different approximations of turbulent transport consider the role of small and large eddies differently. **Local closures**, which neglect the large eddies, are most common. This gives turbulent heat fluxes that flow down the local gradient of potential temperature, analogous to molecular diffusion or conduction (see the Heat and Air Pollution chapters). One such turbulence closure is called **K-theory**.

A **nonlocal closure** alternative that accounts for the superposition of both large and small eddies is called **transilient turbulence theory** (T3). While this is more accurate, it is also more complicated. There are many other closures that have been proposed. K-theory is reviewed here.

18.6.7.2. K-Theory

One approximation to turbulent transport considers only small eddies. This approach, called **K-theory**, **gradient transport theory**, or **eddy-diffusion theory**, models turbulent mixing analogously to molecular diffusion For example, heat flux F_H can be modeled as:

$$F_H = \overline{w'\theta'} = -K \cdot \frac{\Delta\overline{\theta}}{\Delta z} \qquad \bullet(18.41a)$$

This parameterization says that heat flows down the gradient of potential temperature, from warm to cold. The rate of this turbulent transfer is proportional to the parameter K, called the **eddy viscosity** or **eddy diffusivity**, with units $m^2 \cdot s^{-1}$.

Similar expressions can be made for moisture flux as a function of the mean mixing-ratio (r) gradient, or momentum flux as a function of the shear in horizontal wind components (U, V):

$$\overline{w'r'} = -K \frac{\Delta \overline{r}}{\Delta z} \tag{18.41b}$$

$$\overline{w'u'} = -K \frac{\Delta \overline{U}}{\Delta z} \tag{18.41c}$$

$$\overline{w'v'} = -K \frac{\Delta \overline{V}}{\Delta z} \tag{18.41d}$$

K is expected to be larger for more intense turbulence. In the surface layer, turbulence is generated by wind shear. **Prandtl** made a **mixing-length** suggestion that:

$$K = k^2 \cdot z^2 \cdot \left| \frac{\Delta \overline{M}}{\Delta z} \right| \tag{18.42}$$

where k = 0.4 is von Kármán's constant (dimensionless), z is height above ground, and $\Delta M/\Delta z$ is mean shear of the horizontal wind M.

When K-theory is used in the Eulerian heat budget equation, neglecting all other terms except turbulence, the result gives the heating rate of air at height z due to **turbulent flux divergence** (i.e., change of flux with height):

$$\frac{\Delta\overline{\theta}(z)}{\Delta t} = K \cdot \left[\frac{\overline{\theta}(z+\Delta z) - 2\overline{\theta}(z) + \overline{\theta}(z-\Delta z)}{(\Delta z)^2} \right] \tag{18.43}$$

{For those of you who like calculus, the ratio in square brackets is an approximation to the second derivative $[\partial^2 \overline{\theta} / \partial z^2]$. Namely, it is equal to the curvature of the potential temperature vertical profile.} Although the example above was for heat flux, you can also use it for moisture or momentum flux by substituting $\overline{r}$, $\overline{U}$, or $\overline{V}$ in place of $\overline{\theta}$.

K-theory works best for windy surface layers, where turbulent eddy sizes are relatively small. The green arrow in Fig. 18.26a shows that heat flux flows "down" the temperature gradient from warmer to colder potential temperature (orange arrow), which gives a negative (downward) heat flux (Fig 18.26b) in the statically stable surface layer. Fig. 18.25 is also a small-eddy (K-theory-like) illustration.

K-theory does not apply at the solid ground, but only within the air where turbulence exists. For

A SCIENTIFIC PERSPECTIVE • Parameterization Rules

For some situations in science and engineering, the governing equations are not known or are so complicated as to be unwieldy. Nonetheless, an approximation might be good enough to give useful answers.

A **parameterization** is an approximation to an unknown term, made using one or more known terms or factors. Because these approximations do not come from first principles, one or more "fudge factors" are often included in the substitute term to give it the correct behavior and/or order-of-magnitude. These fudge factors are called **parameters**, and are not known from first principles. They must be found **empirically** (from field or laboratory experiments, or from numerical simulations).

Different scientists and engineers can invent different parameterizations for the same unknown term. While none are perfect, each would have different advantages and disadvantages. However, every parameterization must follow certain rules.

The parameterization should:

(1) be physically reasonable,
(2) have the same dimensions as the unknown term,
(3) have the same scalar or vector properties,
(4) have the same symmetries,
(5) be invariant under an arbitrary transformation of coordinate system,
(6) be invariant under an **inertial** or **Newtonian transformation** (e.g., a coordinate system moving at constant speed and direction), and
(7) satisfy the same constraints & budget equations.

Even if the parameterization satisfies the above rules, it often will be successful for only a limited range of conditions. An example from this book is the transport velocity, w_T (see section 10.3.5). One parameterization was developed for statically neutral boundary layers, while a different one worked better for statically unstable mixed layers. Thus, every parameterization should state the limitations on its use, including accuracy and range of validity.

In summary, parameterizations only approximate nature. As a result, they will never work perfectly. But they can be designed to work satisfactorily.

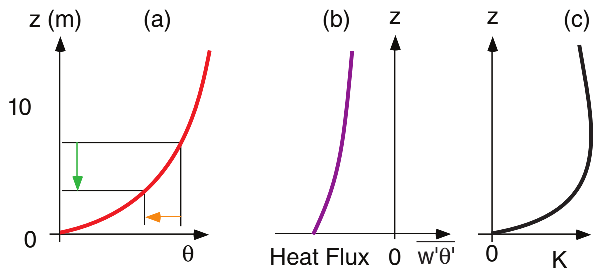


Figure 18.26
Typical profiles of (a) potential temperature; (b) heat flux; and (c) eddy diffusivity in the statically stable surface layer.

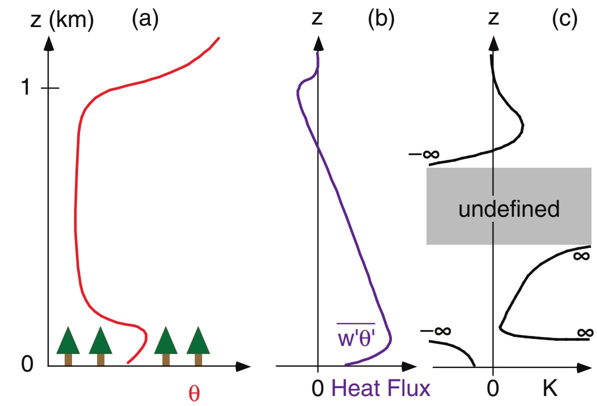


Figure 18.27
The value (c) of K needed to get (b) the observed heat flux F_H from (a) the observed potential temperature (θ) profile over a forest for an unstable ABL. This illustrates that K-theory fails for convective ABLs where turbulence is highly nonlocal.

Sample Application

Instruments on a tower measure θ = 15°C and M = 5 m s^{-1} at z = 4 m, and θ = 16°C and M = 8 m s^{-1} at z = 10 m. What is the vertical heat flux?

Find the Answer:

Given:

z (m)	θ (°C)	M (m s^{-1})
10	16	8
4	15	5

Find: F_H = ? K·m s^{-1}

First, use eq. (18.42), at average z = (10+4)/2 = 7 m

$$K = k^2 \cdot z^2 \cdot \left|\frac{\Delta M}{\Delta z}\right| = [0.4\cdot(7\text{m})]^2 \left|\frac{(8-5)\text{m/s}}{(10-4)\text{m}}\right| = 3.92 \text{ m}^2 \text{ s}^{-1}$$

Next, use eq. (18.41):

$F_H = -K(\Delta\theta/\Delta z) = -(3.92 \text{ m}^2 \text{ s}^{-1})\cdot(16-15°\text{C})/(10-4\text{m})$

= **−0.65 K·m** s^{-1}

Check: Units OK. Physics OK.

Exposition: The negative sign means a downward heat flux, from hot to cold. This is typical for statically stable ABL.

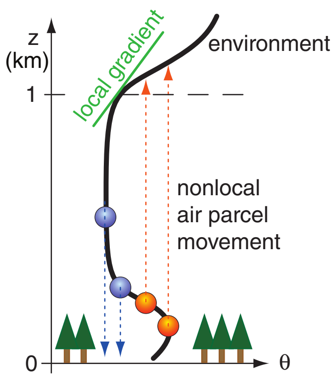


Figure 18.28
Typical environmental potential temperature (θ) profile in the ABL over a forest during daytime. During free convection, buoyancy drives many air parcels to move "nonlocally" across relatively large vertical distances, as illustrated with the dotted lines.

heat fluxes at the surface, use approximations given in earlier in this chapter, and in the Heat chapter.

K-theory has difficulty with convective ABLs and should not be used there. Figs. 18.27a & b illustrate these difficulties, giving typical values in the atmosphere, and the resulting K values computed from eq. (18.42). Negative and infinite K values are unphysical.

18.6.7.3. Nonlocal Closure

Instead of looking at local down-gradient transport as was done in K-theory, you can look at the full range of distances across that air parcels move in turbulent conditions (Fig. 18.28). This is an approach called **nonlocal closure**, and is useful for a statically unstable ABL having free convection. [You might want to review the "parcel (apex) method" in Chapter 5 for determining nonlocal static stability.]

For heat flux at the altitude of the dashed line in Fig. 18.28, K-theory (small-eddy theory) would utilize the local gradient of θ at that altitude, and conclude that the heat flux should be downward and of a certain magnitude. However, if the larger-size eddies are also included (as in nonlocal closure), such as the parcel rising from tree-top level, we see that it is bringing warm air upward (a positive contribution to heat flux). This could partially or completely counteract the negative contribution to flux caused by the local small eddies.

As you can probably anticipate, a better approach would be to consider eddies of all sizes and the associated nonlocal air-parcel movements. One such approach is called **transilient turbulence theory** (T3). This approach uses arrays to account for eddies of each different size, and combines them to find the average effect of all eddy sizes. It is more complex, and will not be described here.

18.7. REVIEW

We live in a part of the atmosphere known as the boundary layer (ABL) — the bottom 200 m to 4 km of the troposphere. Tropospheric static stability and turbulence near the Earth's surface combine to create this ABL, and cap it with a temperature inversion.

Within the ABL are significant daily variations of temperature, winds, static stability, and turbulence over land under mostly clear skies. These variations are driven by the diurnal cycle of solar heating of the ground during daytime and infrared cooling at night.

Above the boundary layer is the free atmosphere, which is not turbulently coupled with the ground (except during stormy weather such as near low pressure centers, fronts, and thunderstorms). Thus, the free atmosphere does not normally experience a strong diurnal cycle.

In the daytime ABL under fair weather conditions (i.e., in anticyclonic or high-pressure regions), vigorous turbulence mixes potential temperature, humidity, wind speed, and pollutants such that they become nearly uniform with height. This turbulence creates a well-mixed layer that grows due to entrainment of free-atmosphere air from above.

At night in fair weather, there is a shallow stable boundary layer near the ground, with a nearly neutral residual layer above. Turbulence is weak and sporadic. Winds often become calm near the surface, but can be very fast a few hundred meters above ground.

The bottom 5 to 10% of the ABL is called the surface layer. Surface drag causes the wind to be zero near the ground, and to increase with height. The shape of this wind profile is somewhat logarithmic, but depends on the roughness of the surface, and on convection.

Turbulence is a quasi-random flow phenomenon that can be described by statistics. Covariance of vertical velocity with another variable represents the vertical kinematic flux of that variable. Heat fluxes, moisture fluxes, and momentum fluxes (stress) can be expressed as such an eddy-correlation statistic.

Velocity variances represent components of turbulence kinetic energy (*TKE*) per unit mass, a measure of the intensity of turbulence. *TKE* is produced by wind shear and buoyancy, is advected from place to place by the mean and turbulent winds, and is dissipated into heat by molecular viscosity.

The relative magnitudes of the shear and buoyant production terms determine whether convection is free or forced. The sum of those terms is proportional to the intensity of turbulence. The ratio gives the flux Richardson number for determining whether turbulence can persist.

Turbulence is so complex that it cannot be solved exactly for each swirl and eddy. Instead, parameterizations are devised to allow approximate solutions for the net statistical effect of all turbulent eddies. Parameterizations, while not perfect, are acceptable if they satisfy certain rules.

One type of local parameterization, called K-theory, neglects the large eddies, but gives good answers for special regions such as the surface layer in the bottom 10% of the atmospheric boundary layer. It is popular because of its simplicity. Another type of parameterization is called transilient turbulence theory (T3), which is a nonlocal closure that includes all eddy sizes. It is more accurate, more complicated, and works well for free convection.

18.8. HOMEWORK EXERCISES

18.8.1. Broaden Knowledge & Comprehension

B1. Access the upper-air soundings every 6 or 12 h for a rawinsonde station near you (or other site specified by your instructor). For heights every 200 m (or every 2 kPa) from the surface, plot how the temperature varies with time over several days. The result should look like Fig. 18.4, but with more lines. Which heights or pressure levels appear to be above the ABL?

B2. Same as the previous question, but first convert the temperatures to potential temperatures at those selected heights. This should look even more like Fig. 18.4.

B3. Access temperature profiles from the web for the rawinsonde station closest to you (or for some other sounding station specified by your instructor). Convert the resulting temperatures to potential temperatures, and plot the resulting θ vs z. Can you identify the top of the ABL? Consider the time of day when the sounding was made, to help you anticipate what type of ABL exists (e.g., mixed layer, stable boundary layer, neutral layer.)

B4. Access a weather map with surface wind observations. Find a situation or a location where there is a low pressure center. Draw a hypothetical circle around the center of the low, and find the average inflow velocity component across this circle. Using volume conservation, and assuming a 1 km thick ABL, what vertical velocity to you anticipate over the low?

B5. Same as previous question, but for a high-pressure center.

B6. Access a number of rawinsonde soundings for stations more-or-less along a straight line that crosses through a cold front. Identify the ABL top both ahead of and behind the front.

B7. Access the current sunrise and sunset times at your location. Estimate a curve such as in Fig. 18.10.

B8. Find a rawinsonde station in the center of a clear high pressure region, and access the soundings every 6 h if possible. If they are not available over N. America, try Europe. Use all of the temperature, humidity, and wind information to determine the evolution of the ABL structure, and create a sketch simi-