CHAPTER 2

AN AIR DISPERSION MODELING PRIMER

2.1 INTRODUCTION

There are many different types of dispersion models, ranging from Gaussian plume models based on atmospheric stability classes to models based on computational fluid dynamics. The air dispersion modeler should be familiar with several of those, with their strengths and weaknesses. The background knowledge needed to build up to that level of expertise takes up a major portion of this book. However, simple air dispersion models can be developed with only a minimum of background knowledge, and we can learn a lot from such models.

When we look at the atmosphere, we see that there are *many factors that influence the behavior of a pollutant plume*: wind speed, atmospheric stability, the occurrence of temperature inversion, plume temperature, plume exit speed, landscape, and obstacles near the pollution source. Except for the last two, all these effects are incorporated in even the most basic dispersion model, the **Gaussian plume model**. This has some important consequences:

- Unless the terrain surrounding the source is complex, Gaussian plume dispersion models can accurately predict concentrations around a source (within a factor of 2).
- Quick calculations with Gaussian plume dispersion models can give you a feel for factors influencing atmospheric dispersion.

For these reasons very simple Gaussian plume models are routinely used for *screening* purposes: When a source of air pollution has been identified, a quick calculation can establish if it is worthwhile to conduct more detailed calculations to estimate the impact of the source. Regulators recommend this approach in their guidelines (e.g., EPA, 2005; Idriss and Spurrell, 2009).

To understand air dispersion, a good knowledge of many meteorological principles is needed. These principles will be discussed in much detail in subsequent chapters. The link between these principles and air dispersion is not always obvious. This primer will help you see that link.

Air Dispersion Modeling: Foundations and Applications, First Edition. Alex De Visscher.

At the end of this primer you will be able to:

- Conduct dispersion calculations for screening a potential source of air pollution
- Understand the effect of the main factors influencing air dispersion
- Understand the main weaknesses of simple models so we can improve on them.

The model outlined in this chapter is similar to ISC3, but with a number of simplifications. The material presented here is similar to introductory texts on air dispersion modeling in environmental engineering textbooks such as Cooper and Alley (2011). Hence, readers familiar with the basics of Gaussian plume modeling may prefer to skip the main part of this chapter and go straight to Section 2.5. If not, the primer will provide a quick sense of accomplishment, as you will be conducting your own air dispersion calculations by the end of this chapter.

2.2 BASIC CONCEPTS OF AIR DISPERSION

When we watch a plume emitted by a stack, we usually observe the following:

- The plume rises and stabilizes at a certain height.
- The plume fans out horizontally and vertically.
- The plume shape fluctuates randomly.

The random fluctuations of a plume cannot be captured by a deterministic model and will not be considered in this primer. However, the *average* pollutant concentration over a given period of time (e.g., 1 h) can be captured. The concepts are shown in Figure 2.1.

In Figure 2.1 the following definitions apply:

h_s = source height (m)
Δh = plume rise (m)
h = effective source height (m)

Both the fanning out of the plume and the random fluctuations are the result of **turbulence** in the wind flow (eddies). Turbulence can be caused by heat released in the atmosphere (thermal turbulence) and by air passing obstacles and roughness of the surface (mechanical turbulence). It follows that *atmospheric dispersion can only be quantified if something is known about the atmosphere* and *the surface*.

Plume rise can be caused by the **momentum** of the plume as it leaves the stack or by the **buoyancy** as hot plumes are lighter than ambient air. Usually, buoyancy is the main driving mechanism for plume rise.

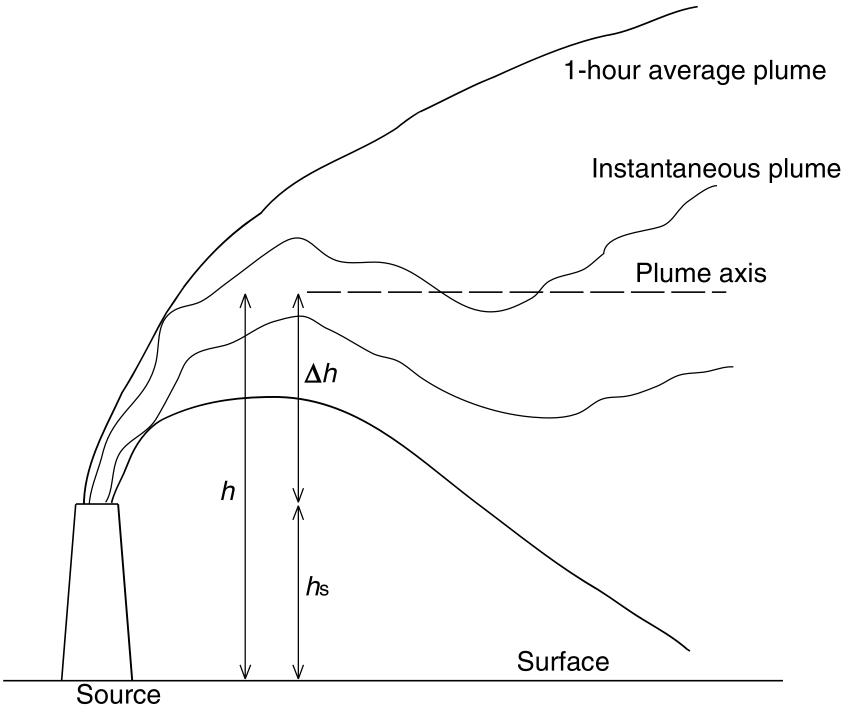

Figure 2.1 Concepts of plume dispersion.

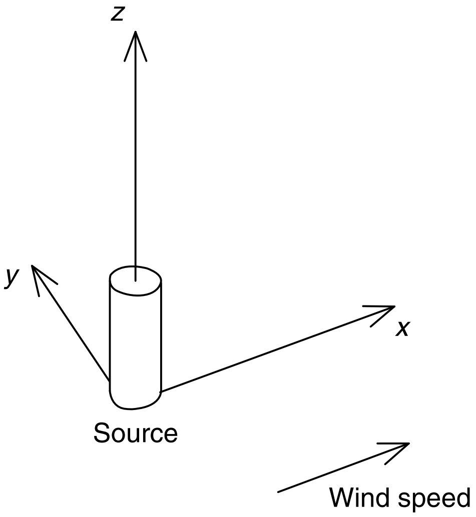

Figure 2.2 Coordinate system in simple Gaussian dispersion models.

A *convention* in Gaussian plume dispersion modeling is the definition of the coordinate axes (see Fig. 2.2):

x = direction of the wind ($x = 0$ at the source; $x > 0$ downwind).

y = horizontal direction perpendicular to the wind ($y = 0$ at the center of the plume; positive on your left when you look downwind).

z = vertical direction ($z = 0$ at the surface and positive above the surface).

In Eulerian air quality modeling we will use a different convention: we define the x coordinate from west to east and the y coordinate from south to north.

The angle of the wind direction itself is also subject to a convention. Typically 0° is associated with a northern wind (i.e., moving south), 90° is associated with an eastern wind, and so forth.

2.3 GAUSSIAN DISPERSION MODEL

2.3.1 Assumptions Underlying the Gaussian Plume Concept

If the fluctuations in plume shape in the *y* and *z* directions are completely random, then the instantaneous concentration profile of a pollutant in a plume will be irregular, while the time-averaged concentration will be a Gaussian distribution in the *y* and *z* directions. This is illustrated in Figure 2.3. Figure 2.4 shows top and side projection of a Gaussian pollutant plume. As a rule of thumb, the plume edge is 2.15 standard deviations from the plume center (Gifford, 1961).

Strictly speaking, the Gaussian plume model is only valid under certain simplifying conditions:

- The plume starts from a mathematical point referred to as a **point source**.
- The source of pollution is constant.
- Wind direction and wind speed are constant in space and time.
- Atmospheric turbulence is constant in space and time.

This means that the Gaussian plume model can only be an approximation, as none of these conditions is ever satisfied. It can be expected that the model is *approximately valid* (within a factor of 2) when the above conditions are approximately valid, and the model becomes less accurate as the real conditions deviate more from these idealized assumptions. Accounting for hills and valleys is especially challenging.

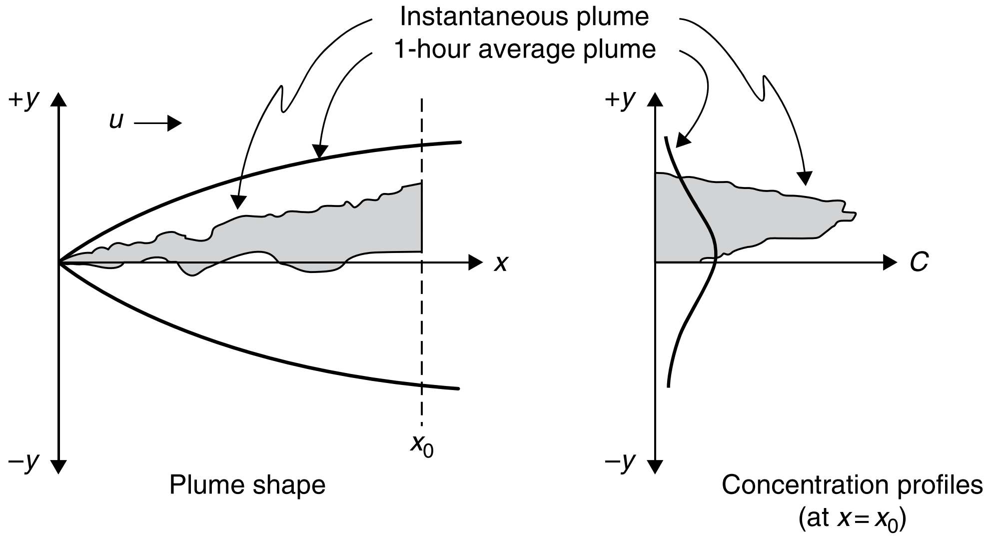

Figure 2.3 Top view of an instantaneous plume and a 1-h average plume and their corresponding concentration profiles (from Cooper and Alley, 2011).

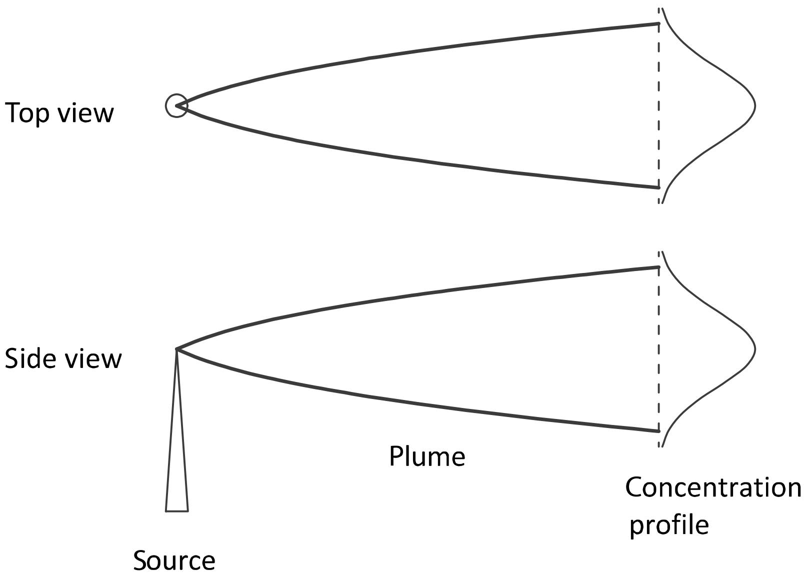

Figure 2.4 Top and side projection of a Gaussian plume, with concentration profiles.

How to handle conditions that deviate from the idealized conditions given above will be discussed in the next chapters.

2.3.2 Quantitative Description

2.3.2.1 Gaussian Plume Equation In the *absence of boundaries*, the equation for pollutant concentrations in Gaussian plumes is as follows:

$$c = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right) \exp\left[-\frac{1}{2}\frac{(z-h)^2}{\sigma_z^2}\right] \tag{2.1}$$

where

c = concentration at a given point (g m^{-3})

Q = emission rate (g s^{-1})

u = wind speed (m s^{-1})

σ_y = dispersion parameter in the horizontal (lateral) direction (m)

σ_z = dispersion parameter in the vertical direction (m)

and x, y, and z have been defined before, as well as the effective source height h. They all have meters as units. Further justification and interpretation of eq. (2.1) is provided in Chapter 6.

Some remarks:

- Because wind speed depends on height, the wind speed *at the effective source height h* should be used for u.
- The spread parameters depend on the distance from the source and on weather conditions.

In practice, *boundaries* usually play a role in air dispersion, in particular the *surface* (i.e., the ground or a water surface). To account for the surface, another assumption is needed. Most pollutants deposit to the surface only slowly, so the conservative assumption that there is no deposition at all is usually made. Hence, the plume behaves as if it *reflects* on the surface. To calculate this effect, an *imaginary* source is defined as shown in Figure 2.5. Equation (2.1) is extended to:

$$C = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\left\{\exp\left[-\frac{1}{2}\frac{(z-h)^2}{\sigma_z^2}\right] + \exp\left[-\frac{1}{2}\frac{(z+h)^2}{\sigma_z^2}\right]\right\} \tag{2.2}$$

Figure 2.6 shows a vertical concentration profile of the direct plume, the reflected plume, and the total plume. The figure clearly demonstrates that it is crucial to take

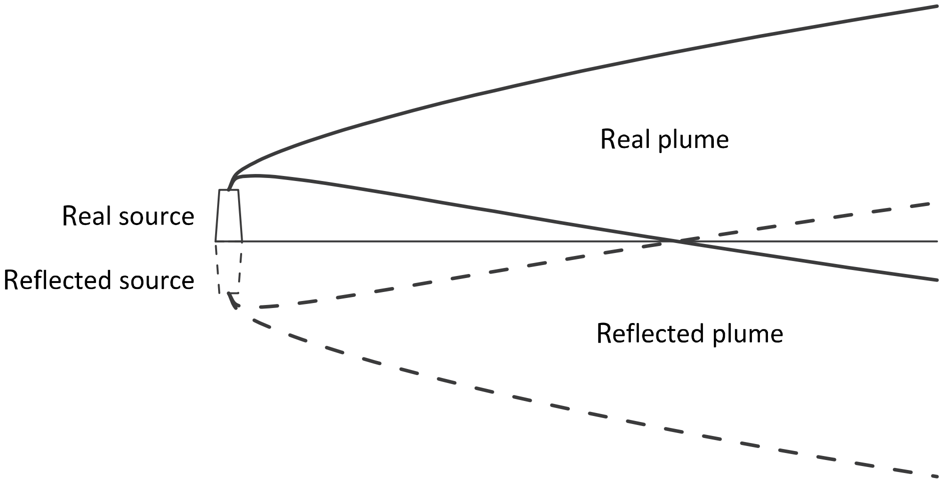

Figure 2.5 Plume reflection on the ground (after Cooper and Alley, 2011).

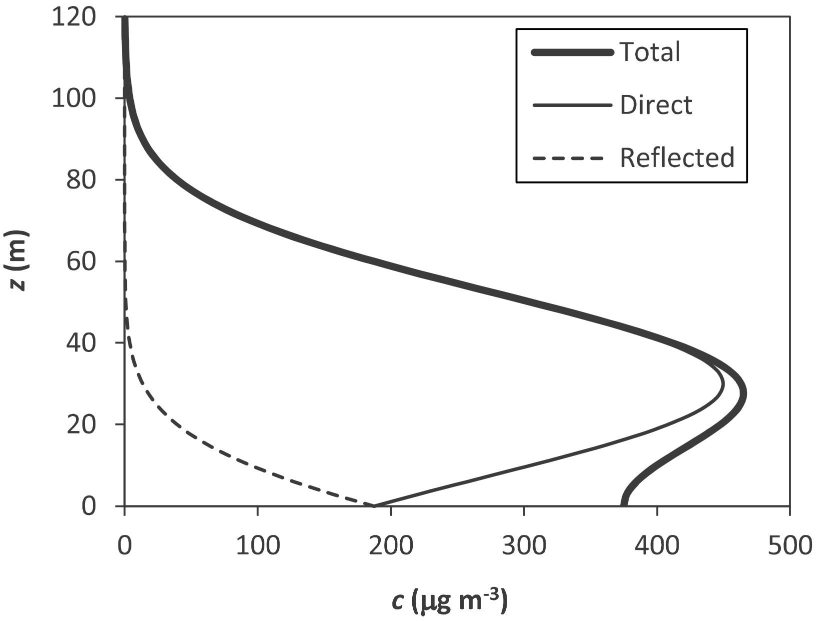

Figure 2.6 Concentration profiles of a direct and a reflected plume.

plume reflection into account. Without the reflection, the ground-level concentration would be underestimated by a factor of 2.

From looking at eq. (2.2), a number of observations can be made directly:

- The pollutant concentration is proportional to the pollutant emission.
- The pollutant concentration is inversely proportional to the wind speed: High pollutant concentrations can be expected at low wind speeds.
- If emission and wind speed affect the plume rise, then the above two statements are only approximations.

Further inferences will be made based on calculated concentration profiles. First, we need to find a way to calculate σ_y and σ_z.

2.3.2.2 Dispersion Parameters—Stability Classes

In order to calculate σ_y and σ_z, we need to know something about the weather conditions at the site of the emission. Until recently, most air dispersion calculations were made based on **stability classes**. Recently, more sophisticated approaches have found their way into the regulatory models. These will be discussed in Chapter 6. At this point, stability classes will suffice. The most commonly used classification of atmospheric stability was developed by Pasquill and Gifford (Pasquill, 1961; Gifford, 1961). They defined six classes, named A through F, with A the most unstable class, D neutral atmosphere, and F the most stable class:

A = Very unstable
B = Moderately unstable
C = Slightly unstable
D = Neutral
E = Slightly stable
F = Stable

Criteria for each class are given in Table 2.1. The criteria for solar radiation are not sharply defined for all possible conditions and require some interpretation by the user. Hence, stability classes are subjective to some degree. As the evaluation of the incoming solar radiation depends on the position of the sun above the horizon, the stability class will depend on the latitude. The key observations to remember from Table 2.1 are the following:

- The atmosphere is unstable on sunny days, neutral on overcast days and nights, and stable on clear nights.
- Increasing the wind speed leads to more neutral conditions.

In the original formulation of the Gaussian dispersion model the dispersion parameters were read from graphs (e.g., Turner, 1970). Several studies have put forward empirical equations to describe these graphs. The most succesful ones are by Briggs (1973), who suggested equations that reflect the autocorrelated stochastic nature of the dispersion process (see Chapters 5 and 6). The equations were calibrated using an extensive data set. The result is shown in Table 2.2 for open (rural)

TABLE 2.1 Criteria for Pasquill–Gifford Stability Classes

	Day			Night	
	Incoming solar radiation			Cloudiness	
u (m s^{-1})[a]	Strong[b]	Moderate[c]	Slight	Cloudy($\geq 4/8$)[d]	Clear ($\leq 3/8$)[e]
<2	A	A–B[f]	B	E	F
2–3	A–B	B	C	E	F
3–5	B	B–C	C	D	E
5–6	C	C–D	D	D	D
>6	C	D	D	D	D

Source: After Turner and Schulze (2007).

[a] Measured at 10-m height.

[b] Clear summer day with sun higher than 60° above the horizon.

[c] Summer day with a few broken clouds or a clear day with the sun 35–60° above the horizon.

[d] Fall afternoon or cloudy summer day with the sun 15–35° above the horizon.

[e] Fractional cloud cover.

[f] Take average dispersion values of two classes.

Note: *Always* use class D for overcast conditions.

TABLE 2.2 Briggs (1973) Equations for Dispersion Parameters in Rural Terrain (Pasquill–Gifford Parameters)[a]

Stability Class	σ_y (m)	σ_z (m)
A	$0.22x(1+0.0001x)^{-0.5}$	$0.2x$
B	$0.16x(1+0.0001x)^{-0.5}$	$0.12x$
C	$0.11x(1+0.0001x)^{-0.5}$	$0.08x(1+0.0002x)^{-0.5}$
D	$0.08x(1+0.0001x)^{-0.5}$	$0.06x(1+0.0015x)^{-0.5}$
E	$0.06x(1+0.0001x)^{-0.5}$	$0.03x(1+0.0003x)^{-1}$
F	$0.04x(1+0.0001x)^{-0.5}$	$0.016x(1+0.0003x)^{-1}$

[a] x is the distance to the source in meters.

TABLE 2.3 Briggs (1973) Equations for Dispersion Parameters in Urban Terrain (McElroy–Pooler Parameters)[a]

Stability Class	σ_y (m)	σ_z (m)
A–B	$0.32x(1+0.0004x)^{-0.5}$	$0.24x(1+0.0001x)^{0.5}$
C	$0.22x(1+0.0004x)^{-0.5}$	$0.2x$
D	$0.16x(1+0.0004x)^{-0.5}$	$0.14x(1+0.0003x)^{-0.5}$
E–F	$0.11x(1+0.0004x)^{-0.5}$	$0.08x(1+0.0015x)^{-0.5}$

[a] x is the distance to the source in meters.

terrain and in Table 2.3 for urban or industrial terrain. The latter were based on the experimental work of McElroy and Pooler (1968).

The equations apply up to a 10-km distance and become increasingly unreliable at larger distances. They are *not recommended* beyond a 30-km distance.

It is generally accepted that these and other calculation schemes of the dispersion parameters represent *10-min. averages*. However, the U.S. Environmental

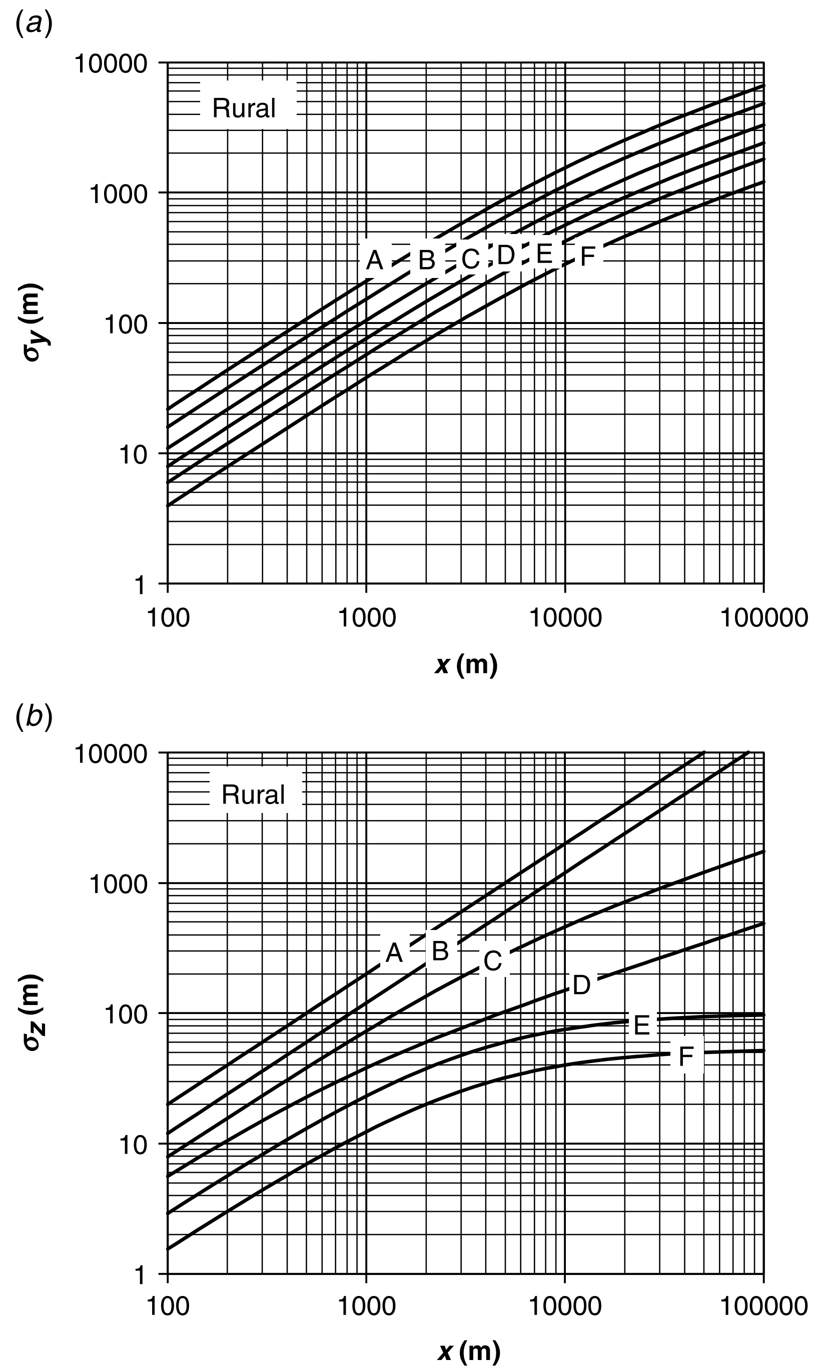

Figure 2.7 (*a*) Horizontal and (*b*) vertical dispersion parameters for rural terrain based on Briggs (1973).

Protection Agency (EPA) treats them as if they are 1-h averages (Beychok, 2005). Consequently, many models, including CALPUFF, use the above equations as if they are hourly averages (Scire et al., 2000a). The significance of this will be discussed further. Note that the Briggs equations, while still the default setting at the time of writing, are by no means the only option in CALPUFF. It is anticipated that the default setting in future releases of CALPUFF will be based on the approaches discussed in Chapters 5 and 6 (J.S. Scire, personal communication).

For quick reference, the Briggs equations are plotted in Figure 2.7 (rural) and Figure 2.8 (urban). The data is included in the enclosed CD (file “Briggs Dispersion Parameters.xlsx”).

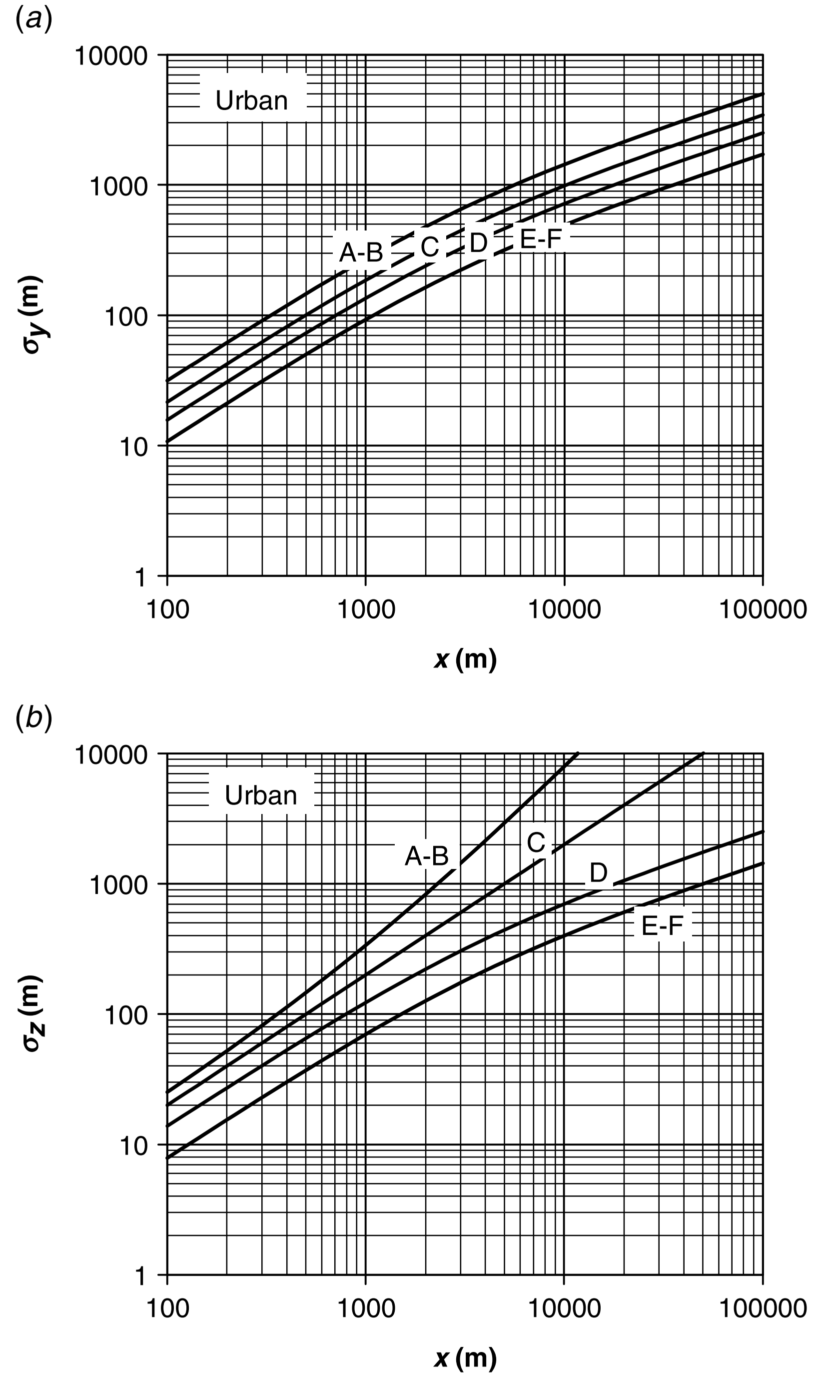

Figure 2.8 (*a*) Horizontal and (*b*) vertical dispersion parameters for urban terrain based on Briggs (1973).

With this introduction, we are ready to attempt our first air dispersion calculation.

Example 2.1. A coal-fired power plant in rural Pennsylvania emits 100 g s^{-1} SO_2 from a stack with height 75 m. Plume rise is 15 m. Wind speed at the effective emission height is 7 m s^{-1}. The weather is overcast. Calculate the SO_2 concentration at ground level 1.5 km downwind from the source at the plume centerline (i.e., directly downwind) and at 100 m lateral distance from the plume centerline. Compare the values with the 24-h ambient air quality standard of 0.14 ppm (assume a temperature of 15 °C, a sea-level corrected barometric pressure of 1 atm, and an altitude of 200 m for the concentration conversion). Do the calculations warrant a further investigation with a more sophisticated model?

Solution. In overcast conditions the stability class is D.

At 1500 m from the source the dispersion parameters are calculated by the Briggs equations for rural conditions:

$$\sigma_y = 0.08 \cdot 1500 \cdot (1 + 0.0001 \cdot 1500)^{-0.5} = 111.9\,\text{m}$$

$$\sigma_z = 0.06 \cdot 1500 \cdot (1 + 0.0015 \cdot 1500)^{-0.5} = 49.9\,\text{m}$$

At the plume centerline ($y = 0$), at ground level ($z = 0$) the concentration is given by

$$\begin{aligned} C &= \frac{100}{2\pi \cdot 7 \cdot 111.9 \cdot 49.9} \exp\left(-\frac{1}{2}\frac{0^2}{\sigma_y^2}\right)\left\{\exp\left[-\frac{1}{2}\frac{(0-90)^2}{49.9^2}\right] + \exp\left[-\frac{1}{2}\frac{(0+90)^2}{49.9^2}\right]\right\} \\ &= 160.3 \times 10^{-6}\,\text{g m}^{-3} \\ &= \mathbf{160.3\,\mu g\,m^{-3}} \end{aligned}$$

At 100-m lateral distance from the plume centerline ($y = 100$ m) the concentration is given by

$$\begin{aligned} C &= \frac{100}{2\pi \cdot 7 \cdot 111.9 \cdot 49.9} \exp\left(-\frac{1}{2}\frac{100^2}{111.9^2}\right)\left\{\exp\left[-\frac{1}{2}\frac{(0-90)^2}{49.9^2}\right] + \exp\left[-\frac{1}{2}\frac{(0+90)^2}{49.9^2}\right]\right\} \\ &= 160.3 \exp\left(-\frac{1}{2}\frac{100^2}{111.9^2}\right) \\ &= \mathbf{107.5\,\mu g\,m^{-3}} \end{aligned}$$

The ambient air quality standard of 0.14 ppm SO_2 is converted to the same units as the air dispersion calculations:

Ideal gas law ($V = nRT/p$): volume of 1 million moles of gas
$= 1{,}000{,}000\,\text{mol} \times 8.314472\,\text{J mol}^{-1}\,\text{K}^{-1} \times (15 + 273.15)\,\text{K}/$
$[101{,}325 \exp(-0.00012 \times 200)\,\text{Pa}]$
$= 24{,}219\,\text{m}^3$

One million moles of gas contains 0.14 mol SO_2 or

$$0.14\,\text{mol} \times 64\,\text{g mol}^{-1} = 8.96\,\text{g } SO_2$$

The concentration is

$$\frac{8.96\,\mathrm{g}}{24{,}219\,\mathrm{m}^3} = 0.000370\,\mathrm{g\,m}^{-3} = 370\,\mu\mathrm{g\,m}^{-3}$$

Without further calculations, we have no indication whether the calculated value of 160.3 μg m^{-3} represents a worst case. Under the given meteorological conditions the highest concentration is found at about 1900 m from the source, and amounts to about 170 μg m^{-3}. If other meteorological conditions can be discarded, then the worst case is more than a factor of 2 below the standard, and more sophisticated calculations are not required. But very likely more unstable conditions and lower wind speeds will lead to higher concentrations, which would prompt more sophisticated calculations.

Comparison of Figure 2.7 with Figure 2.8 shows that dispersion parameters for urban terrain are generally larger than the ones for rural terrain. This is due to the extra turbulence generated by heat (thermal turbulence) and by the shear exerted by the buildings on the air going past. The difference can be a factor of 2 or more for each of σ_y and σ_z. This means that concentration predictions of the two types of terrain can differ by a factor of 4. In practice, all intermediate conditions between fully rural and fully urban exist. Clearly a better parameterization of terrain would be helpful. This problem will be tackled in further chapters.

2.3.2.3 Example Concentration Profiles at Ground Level

For a better understanding of what affects pollutant concentrations, it is useful to look at a number of calculation results.

Figure 2.9 shows typical concentration results versus distance (stability class D, rural terrain, $u = 2\,\mathrm{m\,s}^{-1}$, $Q = 5\,\mathrm{g\,s}^{-1}$, $h = 30\,\mathrm{m}$). The data is included in the enclosed CD ("Concentration vs distance.xlsx"). It is seen that, while the plume axis ($y = 0$, $z = h$) concentration decreases with increasing distance, the ground-level concentration passes through a maximum. Why?

A more unexpected result is that the ground-level concentration exceeds the plume axis concentration at large distances. Why?

Figure 2.10 shows the ground-level pollutant concentration for each of the six stability classes. It can be observed that at sufficiently large distance, the concentrations under unstable conditions are lower than the concentrations at stable conditions. This is because the turbulence dilutes the pollutant. As a result, the impact zone of an air emission is always smaller under unstable conditions than under stable conditions. However, the peak concentration is highest under unstable conditions. This is because turbulence has a larger impact on the vertical dispersion than on the horizontal dispersion. Remember that unstable conditions correspond with sunny days, whereas stable conditions correspond with clear nights. During the day, high pollutant concentrations can occur but they dissipate quickly. At night, pollutants can travel much further than during daytime.

Under most circumstances the difference between concentrations occurring in two adjacent stability classes is less than a factor of 2, but close to the source and far (>5 km) away from the source the difference can be much more. Likewise, away from the plume centerline ($y \neq 0$) the difference between consecutive stability classes

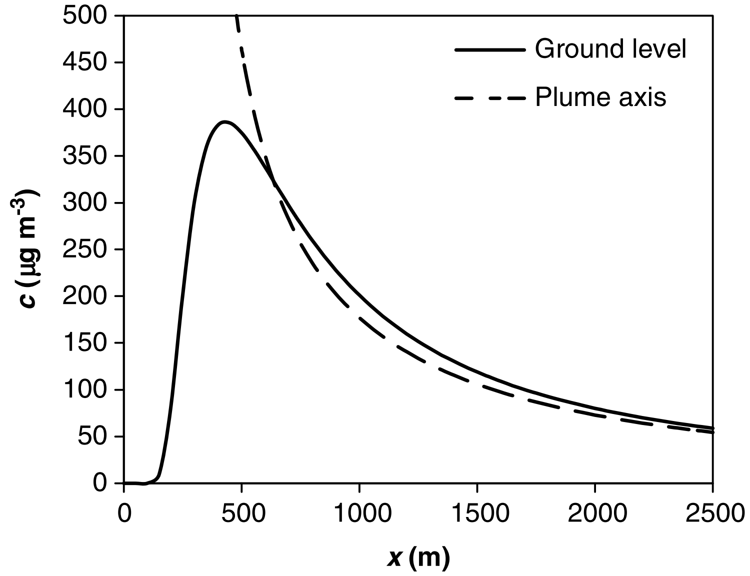

Figure 2.9 Pollutant concentration versus distance from the source.

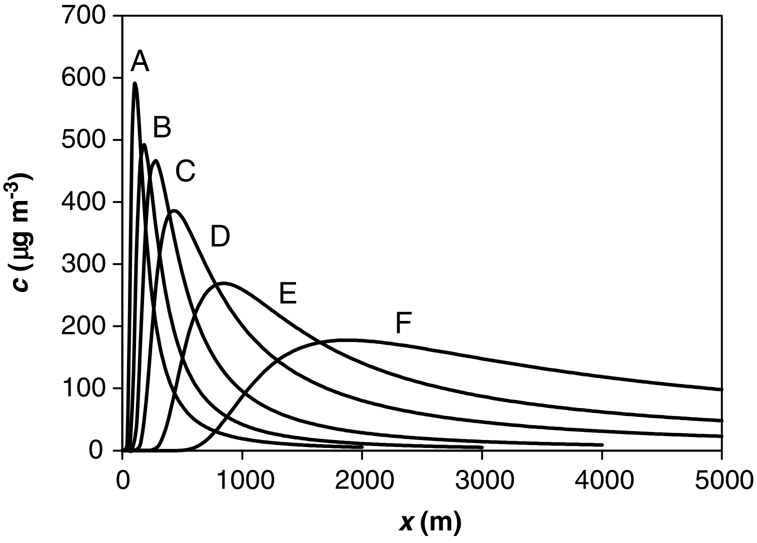

Figure 2.10 Typical ground-level concentrations versus distance for different stability classes.

can be substantially more than a factor of 2. To avoid this, a continuous measure of stability is needed. Such measures will be discussed in Chapters 5 and 6.

Figure 2.11 shows the influence of the effective source height *h* on pollutant concentrations downwind from the source. The conditions are the same as for Figure 2.9. It is clear that increasing the source height decreases the ground-level concentrations. It also increases the distance of the peak concentration. At large distances, the influence of source height is very limited.

2.3.3 Refinements

2.3.3.1 Height Dependence of Wind Speed So far the wind speed at the effective source height was treated as a given. In practice, wind speed is usually known at a different height (at best). Therefore, an equation is needed to relate wind

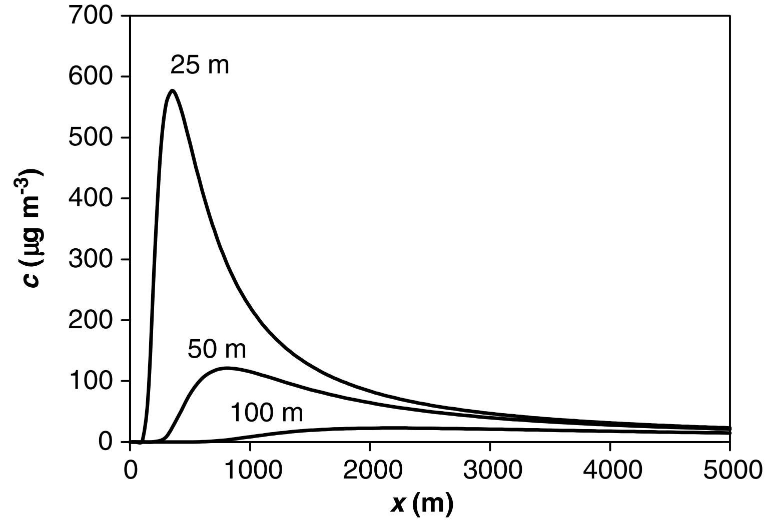

Figure 2.11 Ground-level concentrations versus distance downwind from the source, for three different effective source heights.

TABLE 2.4 Values of *p* for Use in eq. (2.3) to Predict Wind Speed Profiles

Stability Class	*p* for Rural Terrain	*p* of Urban Terrain
A	0.11	0.15
B	0.12	0.15
C	0.12	0.20
D	0.17	0.25
E	0.29	0.40
F	0.45	0.60

Source: Beychok (2005).

speeds at different heights. A sophisticated treatment of wind velocity profiles will be given in Chapter 5. For now an empirical equation will suffice. The following equation is commonly used:

$$u_2 = u_1 \left(\frac{z_2}{z_1} \right)^p \tag{2.3}$$

where u_1 and u_2 are wind speeds 1 and 2, and z_1 and z_2 are heights 1 and 2. There is no agreement in the literature concerning recommended values for p. Based on a comparison of Arya (1999), Scire et al. (2000b), Cooper and Alley (2011), Beychok (2005), on estimates based on Seinfeld and Pandis (2006), and on comparison with more sophisticated meteorological theory, the values given in Table 2.4 are put forward. They are values of Beychok, 2005, based on Touma (1977) and Petersen (1978), for rural and urban terrain, respectively.

Calculated wind speed profiles for urban terrain are shown in Figure 2.12 at constant wind speed at a 10-m height. It is clear that the most pronounced wind speed profiles are found in a stable atmosphere.

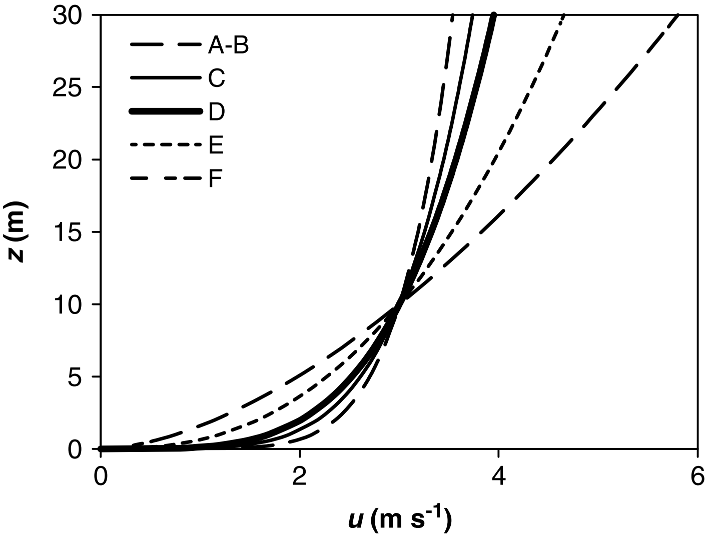

Figure 2.12 Calculated wind speed profiles in urban terrain. $u_{10} = 3\,\mathrm{m\,s^{-1}}$.

Example 2.2. Assume that the wind speed in Example 2.1 was measured at a 10-m height, not at the effective source height. How would that affect the calculation?

Solution. For stability class D in rural terrain a value for p of 0.17 is used. This leads to the following wind speed at a 90-m height:

$$\begin{aligned} u &= 7\,\mathrm{m\,s^{-1}} \cdot (90/10)^{0.17} \\ &= 10.17\,\mathrm{m\,s^{-1}} \end{aligned}$$

This reduces the ground-level centerline concentration at a 1.5-km distance from $160.3\,\mu\mathrm{g\,m^{-3}}$ to $110.3\,\mu\mathrm{g\,m^{-3}}$, and at a 100-m lateral distance from that point from $107.5\,\mu\mathrm{g\,m^{-3}}$ to $74.0\,\mu\mathrm{g\,m^{-3}}$. It follows that height corrections for wind speed can be substantial.

2.3.3.2 Temperature Inversion Layer

A temperature inversion layer is a layer in the atmosphere where the temperature increases with height. Such layers are extremely stable, and act as an effective barrier to further dispersion. Most daytime atmospheres are capped by a temperature inversion, confining dispersion to a mixed layer (or mixing layer) with a thickness ranging from a few hundreds of meters to a few kilometers. In the nighttime atmosphere the temperature inversion usually reaches the surface, and the mixing layer is defined by mechanical mixing effects.

When the atmosphere has a well-defined mixing layer, this needs to be accounted for in the calculation of air dispersion. This is possible by considering the inversion layer capping the mixing layer as another reflecting layer. Hence, the plume is represented as trapped between two reflecting layers, as shown in Figure 2.13.

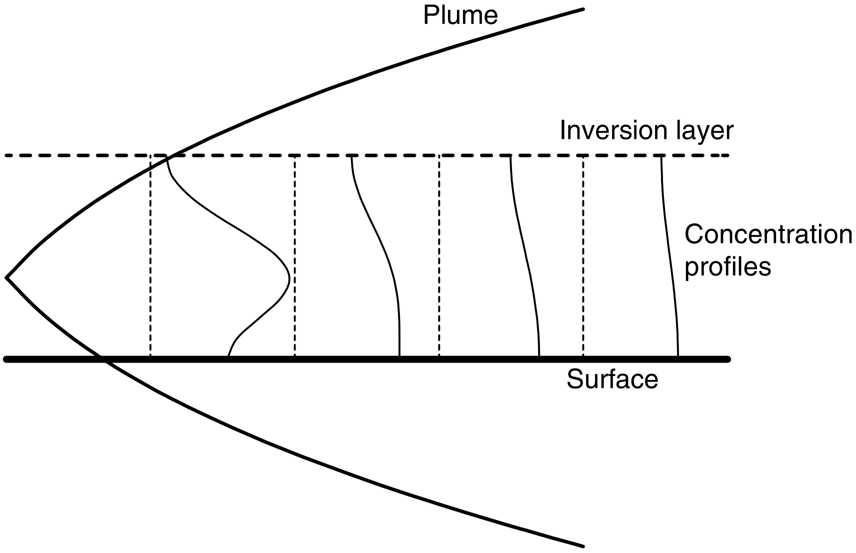

Figure 2.13 Plume dispersion between the Earth's surface and an elevated temperature inversion (after Cooper and Alley, 2011).

In this case, the concentration in the plume can be described by the following equation:

$$C=\frac{Q}{2\pi u\sigma_y\sigma_z}\exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\sum_{j=-\infty}^{+\infty}\left\{\exp\left[-\frac{1}{2}\frac{\left(z-h+2jh_{\mathrm{mix}}\right)^2}{\sigma_z^2}\right]+\exp\left[-\frac{1}{2}\frac{\left(z+h+2jh_{\mathrm{mix}}\right)^2}{\sigma_z^2}\right]\right\} \tag{2.4}$$

where h_{mix} (m) is the distance of the temperature inversion from the ground, also known as the height of the mixed layer, or the height of the mixing layer. After a few reflections the plume can be considered well mixed. In practice, eq. (2.4) can be limited to $j=-1$, 0, +1 to within 2% for values of σ_z up to h_{mix}. Once σ_z exceeds h_{mix}, the concentration can be approximated to within 1.5% by the following approximate equation, which assumes complete mixing in the vertical direction up to h_{mix}:

$$C=\frac{Q}{\sqrt{2\pi}u\sigma_y h_{\mathrm{mix}}}\exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right) \tag{2.5}$$

CALPUFF uses this equation once $\sigma_z > 1.6\ h_{\mathrm{mix}}$. From eq. (2.5) we can see that $C \sim 1/h_{\mathrm{mix}}$ under these conditions. Hence, *good estimates of* h_{mix} *are important when a plume is trapped under an inversion layer*. Ways to estimate h_{mix} when no measured value is available are presented in Chapter 5.

2.4 PLUME RISE

2.4.1 Plume Rise Correlations

There are two possible reasons why a plume rises when it leaves the stack:

- Because of its buoyancy (in case of hot exhaust gases)
- Because of its momentum (in case of high velocity)

The momentum contribution to plume rise is usually small and will be ignored here.

Buoyancy is usually the result of a temperature difference, but it can also occur when the waste gas has a molar mass that deviates considerably from the molar mass of air. However, that would mean that the pollutants are highly concentrated. In practice, regulators will not allow emissions in such conditions, and waste gas treatment will be fairly simple. Hence, this case will not be considered here. It is usually reasonable to assume that the waste gas has the same molecular weight as air. Accounting for buoyancy due to molecular weight differences can be important in the case of accidental releases and is especially worrisome when the gas is heavier than air. The EPA model SLAB (Ermak, 1990; http://www.epa.gov/scram001/dispersion_alt.htm) is specifically designed for that purpose.

Several equations have been proposed to predict plume rise. Unfortunately, the predictions of the different models are more than a factor of 10 apart (Briggs, 1975). Of these, Briggs (1968) developed the most well-conceived equations. They are used in many regulatory models.

First, the buoyancy flux parameter F_b is defined:

$$F_b = \left(1 - \frac{\rho_s}{\rho}\right) g r_s^2 w_s \tag{2.6}$$

with ρ_s the density of the stack gas, ρ the density of the surrounding air, g the acceleration due to gravity (9.80665 m s^{-2}), r_s (m) the stack radius, and w_s (m s^{-1}) the stack gas velocity in the vertical direction.

The *transitional* plume rise is the local plume rise, before the plume has reached its maximum height. It is given by the following equation (Briggs, 1972):

$$\Delta h = \frac{1.6 F_b^{1/3} x^{2/3}}{u} \tag{2.7}$$

where x (m) is the distance downwind from the source, and u is the wind speed (m s^{-1}). However, plumes do not rise indefinitely but stabilize at a certain height, the *final* plume rise height. This height is achieved at a distance x_f (m) from the source (Briggs, 1975):

$$x_f = 49 F_b^{5/8} \qquad \text{for} \quad F_b < 55\,\text{m}^4\,\text{s}^{-3} \tag{2.8}$$

$$x_f = 119F_b^{2/5} \quad \text{for} \quad F_b > 55\,\text{m}^4\,\text{s}^{-3} \tag{2.9}$$

Note that eqs. (2.8) and (2.9) are dimensionally not homogeneous and are only valid when metric units are used. At distances greater than x_f, the plume rise is assumed constant and given by

$$\Delta h = \frac{1.6F_b^{1/3}x_f^{2/3}}{u} \tag{2.10}$$

Example 2.3. Calculate the plume rise at a 1000-m distance of a waste gas stream leaving a stack at 100 °C into an atmosphere at 25 °C and 100 kPa. The flow rate is $20\,\text{m}^3\,\text{s}^{-1}$; the stack diameter is 2 m. Wind speed at the stack exit is $3\,\text{m}\,\text{s}^{-1}$.

Solution. First, we check if the plume rise is transitional or final at $x = 1000\,\text{m}$. For that we need F_b. The variables of eq. (2.7) are

$$\rho = 1.170\,\text{kg}\,\text{m}^{-3} \quad (\text{see Problem 8})$$

$$\rho_s = \rho \times 298.15/373.15 = 0.935\,\text{kg}\,\text{m}^{-3}$$

$$g = 9.80665\,\text{m}\,\text{s}^{-2}$$

$$r_s = 1\,\text{m}\,(\text{diameter}/2)$$

$$w_s = Q/A_s$$

where

Q = volumetric flow rate ($20\,\text{m}^3\,\text{s}^{-1}$)
A_s = stack cross-sectional area ($= \pi r_s^2 = 3.1416\,\text{m}^2$)

Hence $w_s = 6.366\,\text{m}\,\text{s}^{-1}$.

Substitution into eq. (2.7) yields

$$F_b = 12.55\,\text{m}^4\text{s}^{-3}$$

This is substituted into eq. (2.8) ($F_b < 55\,\text{m}^4\,\text{s}^{-3}$):

$$x_f = 238.1\,\text{m} < x$$

It follows that the final plume rise is reached. Equation (2.10) is used, with $u = 3\,\text{m}\,\text{s}^{-1}$:

$$\boldsymbol{\Delta h = 47.6\,\text{m}}$$

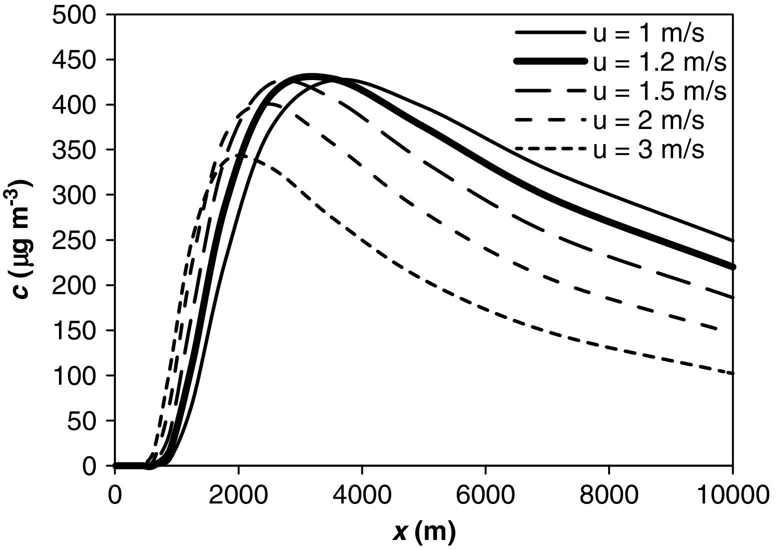

Figure 2.14 Concentration at ground level versus distance from the source at different wind speeds. Critical wind speed is about 1.2 m s^{-1}.

Plume rise is a complex process and depends on different factors that in turn depend on atmospheric conditions. Hence, one set of equations cannot cover all cases, and eqs. (2.7) and (2.10) should be used with care. Here are some issues:

- When the wind speed is low, eq. (2.7) predicts infinite plume rise. This is unrealistic.
- The equations are not designed for strongly unstable or strongly stable conditions.
- The equations do not account for momentum-dominated plume rise.

These issues will be considered in Chapter 7.

2.4.2 Critical Wind Speed

From eq. (2.2) it is clear that the concentration of a pollutant in a plume *decreases* with increasing wind speed. On the other hand, eq. (2.7) shows that the plume rise decreases with increasing wind speed, which leads to a *increase* of the concentration with increasing wind speed. The overall result of these counteracting effects is that the concentration passes through a maximum at a certain wind speed. This wind speed is known as the **critical wind speed**. It can be determined by trial and error. To illustrate the principle of critical wind speed, ground-level concentrations were calculated at different wind speeds for a source with considerable plume rise ($h_s = 75$ m, $Q = 100$ g s^{-1}, rural, stability class D, $F_b = 4$ $m^4 s^{-3}$). The result is shown in Figure 2.14. Here the critical wind speed is around 1.2 m s^{-1}. The file that generated the figure and calculated the critical wind speed ("Figure 2.14. Critical wind speed. xlsx") is included on the enclosed CD. The critical wind speed was calculated with the Solver function (menu Data → Solver). In the file, the Solver is configured

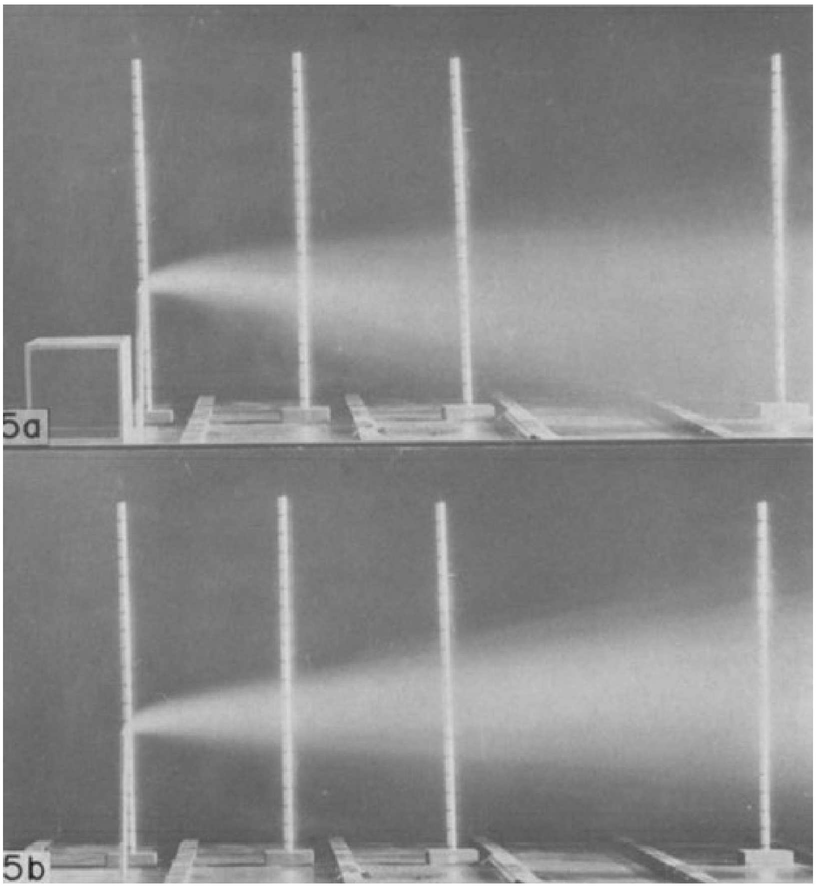

Figure 2.15 Plume dispersion in a wind tunnel experiment with downwash (top) and without downwash (bottom). [Reprinted from Huber and Snyder (1982) with permission from Elsevier.]

to maximize the cell I42 (concentration) by changing cells I4 (wind speed) and A42 (distance). A wind speed of 1.21096 m s^{-1} is obtained, with a maximum concentration of 432.5773 μg m^{-3} at a distance 3157.663 m from the source.

The critical wind speed is a crucial concept in screening calculations. It represents the worst-case scenario for the impact of an emission on ambient concentrations.

2.4.3 Rules of Thumb

The plume rise equations discussed above are only valid when there is no interference between the plume and surrounding buildings. A well-known interference is **downwash**: the plume gets trapped in the wake of a building or is dragged down by the air that follows the wake. An illustration of downwash from a wind tunnel experiment is shown in Figure 2.15 (Huber and Snyder, 1982).

Downwash decreases the effective source height and increases ground-level concentrations. To avoid downwash, the following rules of thumb are generally accepted:

- The stack should be at least 2.5 times as tall as the tallest of the nearest buildings.
- The exit velocity of the exhaust gas should be at least 1.5 times the maximum expected wind speed.

These criteria cannot always be met. For instance, the second criterion can lead to unrealistically high pressure drops in the stack. Sometimes this criterion can be met by narrowing the stack at the top only, thus limiting the friction-based pressure drop. The main reason to impose this criterion is to avoid that the plume gets trapped in the wake of the stack itself (**stack downwash**).

2.5 NEED FOR REFINEMENTS TO THE BASIC GAUSSIAN PLUME DISPERSION MODEL

From the above sections it will be clear that the basic Gaussian dispersion model is far from perfect. Here is a list of issues that need to be resolved in order to obtain a universally applicable dispersion model:

- Sources that are variable in time
- Changing wind speed and wind direction
- Incorporating landscapes (hills, valleys) in dispersion models
- Sources close to the ground, where the wind speed profile is pronounced
- Vertical turbulence profile
- Plume deposition
- Chemical reaction
- Avoiding the use of dispersion classes
- Terrain characterization that is more refined than rural–urban settings
- Better wind speed profiles
- Ways to estimate the height of the mixing layer
- More universal methodology to estimate plume rise

All these issues will be addressed in Chapters 5–11.

PROBLEMS

1. Show that the units of eq. (2.1) are consistent. (*Hint*: exponentials are always dimensionless.)

2. Simplify eq. (2.2) for the case of ground-level concentrations. Simplify further for locations exactly downwind from the source (the plume centerline).

3. What is the highest possible elevation of the sun above the horizon in the area where you live?

4. What is the highest possible latitude for stability class A to occur?
5. Compare the peak concentrations in Figure 2.11. What ratio are they? What is the ratio you would expect?
6. Based on eq. (2.3) and Table 2.4, how does the wind speed profile compare with the velocity profile of a fluid in a pipe?
7. Show that the units of F_b are $m^4\ s^{-3}$.
8. Calculate the density of air at standard ambient temperature and pressure (SATP) (25 °C, 100,000 Pa). The molar mass of air is $29\,g\,mol^{-1}$. Use the ideal gas law.
9. A stack with a height of 50 m emits $20\,g\,s^{-1}$ of NO_2. Plume rise is negligible. The wind speed is $2.5\,m\,s^{-1}$. Stability class is C. The terrain is rural. Make a plot of the downwind NO_2 concentration at ground level at the centerline as a function of distance from the stack. Determine the maximum concentration and the distance where this maximum is observed. Make a plot of the ground-level NO_2 concentration versus distance from the centerline on a transect through the concentration maximum (i.e., a lateral, or cross-wind concentration profile).
10. As a result of odor complaints, a "nasal ranger" is sent to the site of the odor for an investigation. The nasal ranger reports a distinct H_2S smell downwind of a gas plant, up to about 5 km downwind of the plant. The investigation was done on a cloudy fall day at 10 °C, with a wind speed at a 10-m height of $7\,m\,s^{-1}$ and a barometric pressure of 88 kPa. Under lab conditions the nasal ranger could smell H_2S concentrations down to 1 ppb. The gas plant is located in a rural area. All the waste gas is emitted through a 25-m-high stack with inside diameter at the top of 1 m. The plant emits $7\,m^3$ (STP) gas per second containing mainly air, at a temperature of 70 °C. Estimate the H_2S emission based on this information.
11. In an experiment to characterize a particulate matter emission, an array of monitors is placed 200 m downwind from the source to form a cross-wind transect. The monitors are essentially at ground level. The terrain is rural. The effective source height, based on visual inspection of the plume, is determined to be 10 m. The wind speed at a 10-m height is $2\,m\,s^{-1}$. The average concentration measured at the monitors are the following:

y (m)	*c* ($\mu g\,m^{-3}$)
–50	0
–40	2
–30	20
–20	120
–10	350
0	500
10	340
20	125
30	25
40	1.5
50	0

Determine the stability class that corresponds best with the data, as well as the emission.

MATERIALS ONLINE

- “Briggs dispersion parameters.xlsx”: data to Figures 2.7 and 2.8
- “Concentration vs distance.xlsx”: data to Figure 2.9
- “Figure 2.14. Critical wind speed.xlsx”: data to Figure 2.14, with calculation of the critical wind speed

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